ACCELERATED MATHEMATICS

CHAPTER 9

GEOMETRIC PROPERTIES

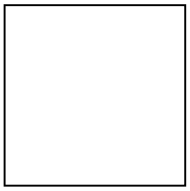
PART I

TOPICS COVERED:

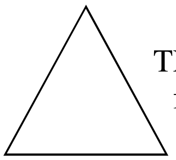
- Geometry vocabulary
- Similarity and congruence
- Classifying quadrilaterals
- Transformations (translations, reflections, rotations, dilations)

Geometry is the area of mathematics that deals with the properties of points, lines, surfaces, and solids. It is derived from the Greek "geometra" which literally means earth measurement.

GEOMETRIC PSYCHOLOGY

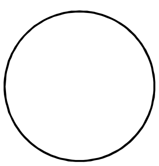


SQUARE – hard worker, likes structured and organized environment, loves data, dependable, tenacious, likes to do the job themselves, likes things in writing, makes sure things get done well, likes lots of details, will not tolerate sloppy work

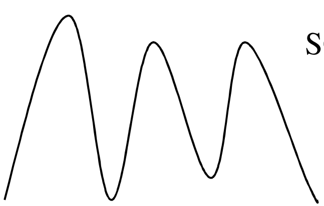


TRIANGLE – leader, very focused, loves recognition, very sure people, outspoken, very focused on goal at hand, loves lists and sticky notes, independent, likes to do his/her own thing, always get the best deals

RECTANGLE – sick of being a square and reaching upward like a triangle, excited, unpredictable, excellent student, less frozen than other students, team players, thinks well in groups



CIRCLE – likes harmony, fun, nurturing, caretaker, loves people with problems so that they can help them solve problems, best listener and best communicator, has good gut ideas, trustworthy, cannot stand conflict, , have a hard time saying no, has many friends

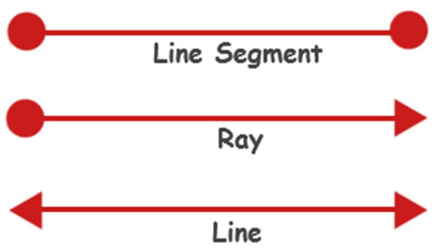


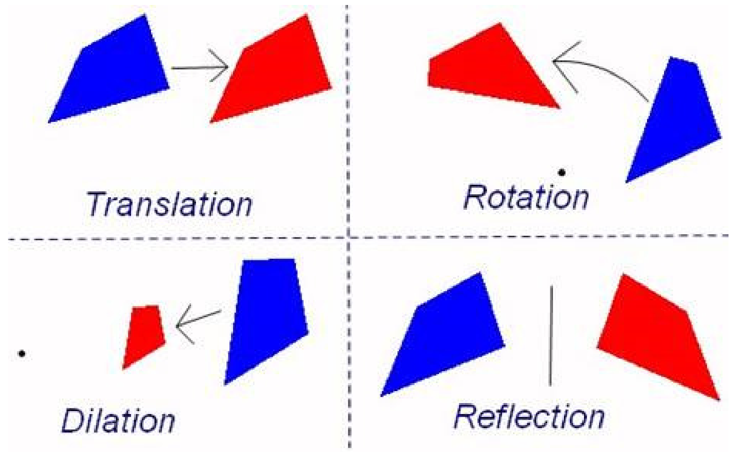
SCRIBBLE – open-ended, most creative, highly conceptual broad ideas, asks "what if" a lot, future oriented, not a detailed person, has lots of ideas both good and bad, good trouble shooter, has a short attention span

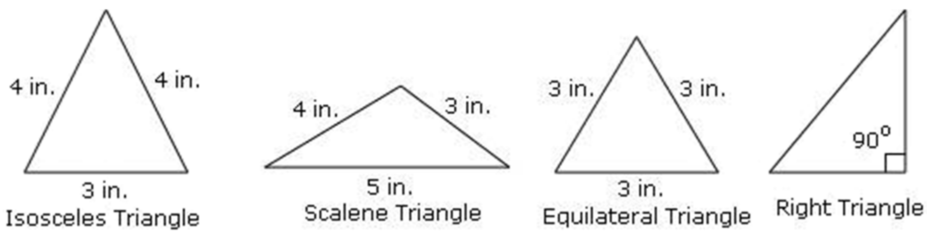
Geometry Puns/Jokes Name:

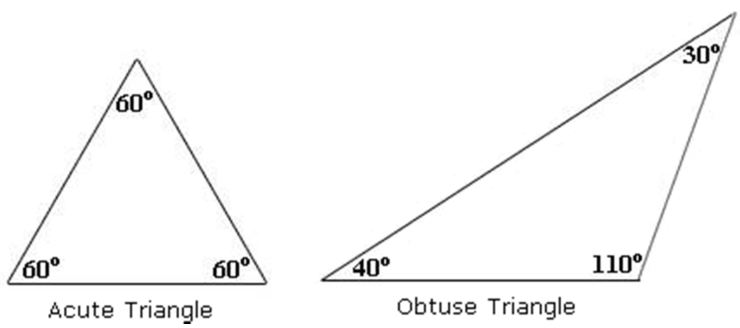
1.	A broken angle	Rectangle
2.	Place where people are sent for committing crimes	Prism
3.	When you have more than one L	Parallel
4.	The opposite of telling the truth	Line
5.	A pretty vertex	Acute angle
6.	What the engineer telegraphed ahead	Translate
7.	An angle that is never wrong	Right angle
8.	Used to tie up packages	Chord
9.	That man does not talk plainly	Ellipse
10.	What girls want to find at the beach	Tangent
11.	They voted "yes" on farm machinery	Protractor
12.	Mathematicians' dessert	Pie
13.	A sharp weapon	Sphere
14.	What little acorns say when they grow up	Geometry
15.	The one in charge	Ruler
16.	What the professor did with the letter he carried for a week before mailing	Postulate
17.	What the man did when his mother-in-law wanted to go home	Center
18.	What a person should do when it rains	Coincide
19.	The way the poet wrote her love letters	Inverse
20.	A missing parrot	Polygon
21.	What he said when the witch doctor removed the curse	Hexagon

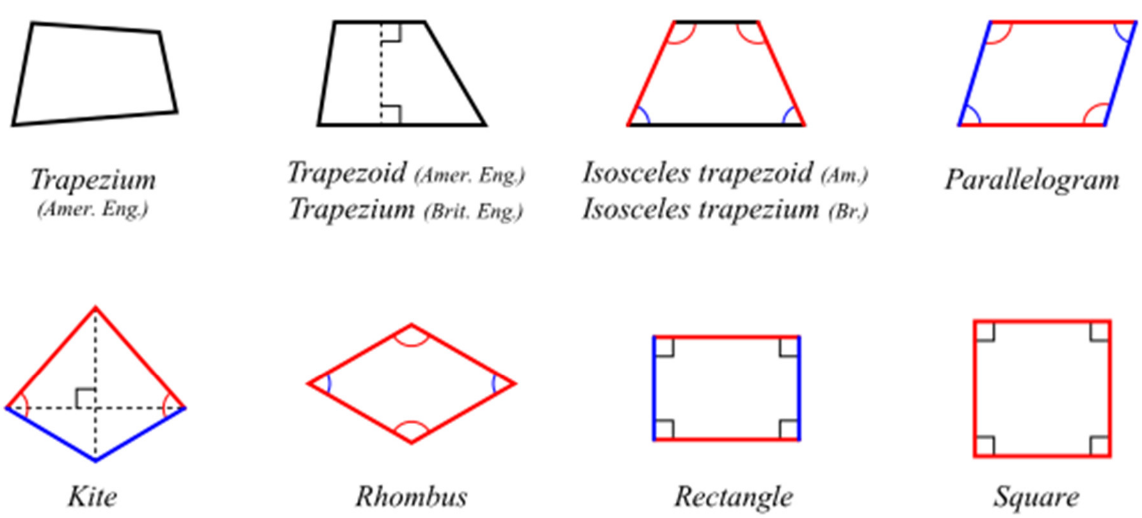
Dictionary of Geometry Name:



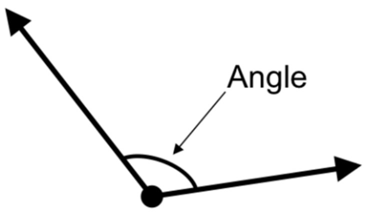


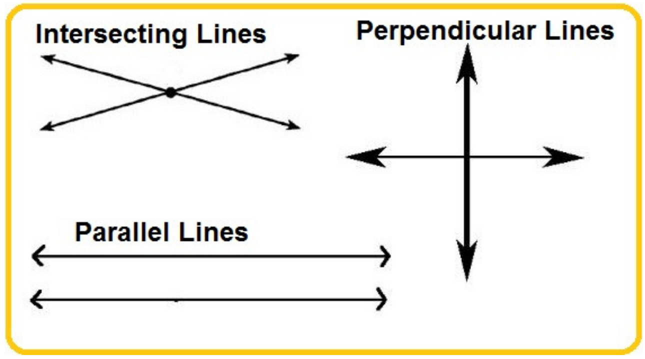


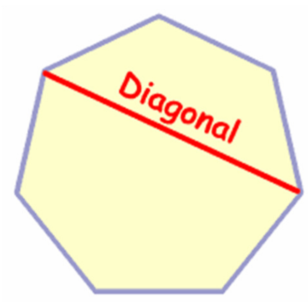


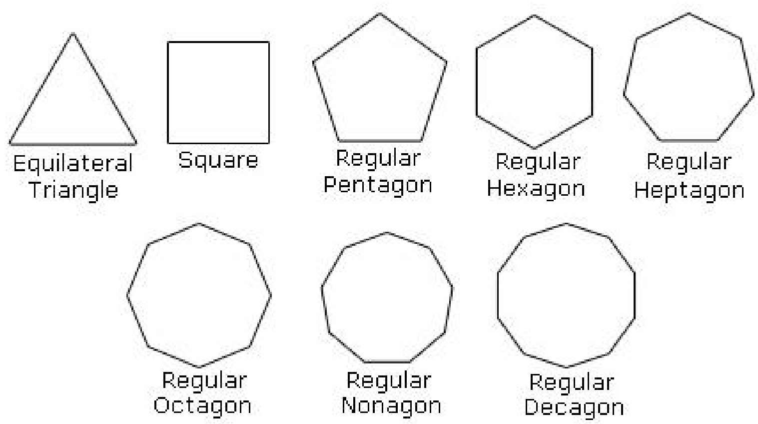


Dictionary of Geometry Name: ______________________









SIDES	NAME		SIDES	NAME
1	monogon		21	icosikaihenagon
2	digon		22	icosikaidigon
3	trigon or triangle		23	icosikaitrigon
4	tetragon or quadrilateral		24	icosikaitetragon
5	pentagon		25	icosikaipentagon
6	hexagon		26	icosikaihexagon
7	heptagon or septagon		27	icosikaiheptagon
8	octagon		28	icosikaioctagon
9	enneagon or nonagon		29	icosikaienneagon
10	decagon		30	triacontagon
11	hendecagon		31	tricontakaihenagon
12	dodecagon		40	tetracontagon
13	triskaidecagon		41	tetracontakaihenagon
14	tetrakaidecagon or tetradecagon		50	pentacontagon
15	pentakaidecagon or pentadecagon		60	hexacontagon
16	hexakaidecagon or hexadecagon		70	heptacontagon
17	heptakaidecagon		80	octacontagon
18	octakaidecagon		90	enneacontagon
19	enneakaidecagon		100	hectogon or hecatontagon
20	icosagon		1000	chiliagon
			10000	myriagon

Activity 9-1: **Vocabulary Match** Name:

Section 1: Polygons

Word bank: Triangle Decagon Nonagon Circle Octagon Quadrilateral Hexagon Pentagon Heptagon Regular polygon Polygon	A geometric figure with 3 or more sides and angles	1. polygon
	A polygon with 3 sides	2. triangle
	A polygon with 4 sides	3. quadrilateral
	A polygon with 5 sides	4. pentagon
	A polygon with 6 sides	5. hexagon
	A polygon with 7 sides	6. heptagon
	A polygon with 8 sides	7. octagon
	A polygon with 9 sides	8. nonagon
	A polygon with 10 sides	9. decagon
	The set of all points in a plane that are the same distance from a given point (hint: not a polygon)	10. circle
	A polygon with all sides congruent and all angles congruent	11. regular polygon

Section 2: Four sided polygons (Quadrilaterals)

Word bank: Trapezoid Parallelogram Rectangle Rhombus Square	A parallelogram with 4 right angles and 4 congruent sides	12. square
	A parallelogram with 4 right angles (sides may or may not be congruent)	13. rectangle
	A parallelogram with 4 congruent sides (any size angles)	14. rhombus
	A quadrilateral with exactly one pair of opposite sides parallel (any size angles)	15. trapezoid
	A quadrilateral with opposite sides parallel and opposite sides congruent	16. parallelogram

Section 3: Shape movement

Word bank: Transformation Reflection Rotation Translation Dilation	Any kind of movement of a geometric figure	17. transformation
	A figures that **slides** from one location to another without changing its size or shape	18. translation
	A figure that is **turned** without changing its size or shape	19. rotation
	A figure that is **flipped** over a line without changing its size or shape	20. reflection
	A figure that is enlarged or reduced using a scale factor	21. dilation

Section 4: Angles

Word bank: Angle Acute angle Right angle Straight angle Obtuse angle Vertex Diagonal	An angle that is exactly $180°$	22. straight angle
	An angle that is less than $90°$	23. acute angle
	The point of intersection of two sides of a polygon	24. vertex
	An angle that is between $90°$ and $180°$	25. obtuse angle
	An angle that is exactly $90°$	26. right angle
	A segment that joins two vertices of a polygon but is not a side	27. diagonal
	A figure formed by two rays that begin at the same point	28. angle

Section 5: Figures and Angles

Word bank: Congruent figures Similar figures Line of symmetry Complementary angles Supplementary angles	Angles that add up to $90°$	29. complementary
	Angles that add up to $180°$	30. supplementary
	Figures that are the same size and same shape	31. congruent figures
	Figures that are the same shape and may or may not have same size	32. similar figures
	Place where a figure can be folded so that both halves are congruent	33. line of symmetry

Section 6: Lines

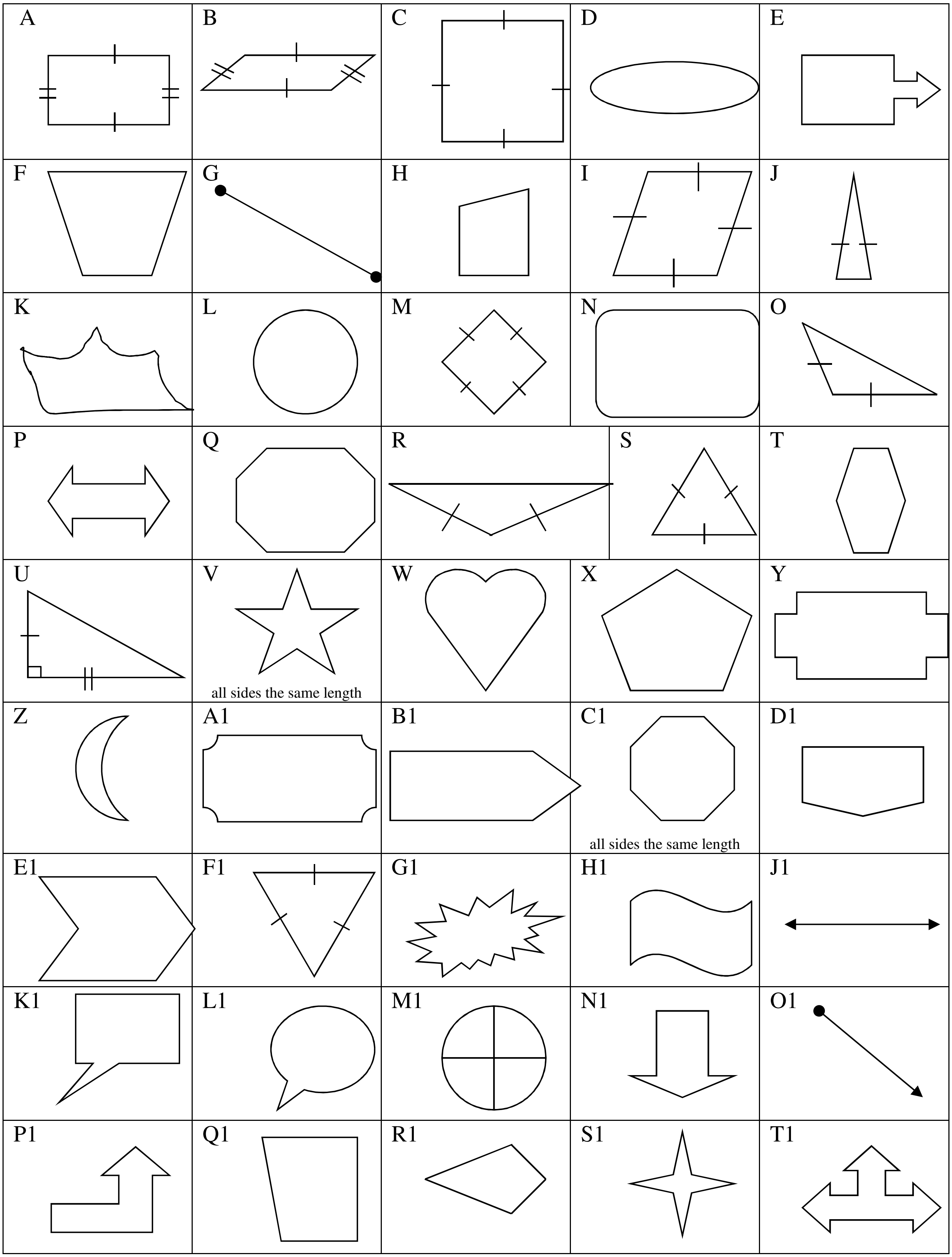
Word bank: Perpendicular line Ray Line Intersecting lines Parallel lines Line segment Point Plane	An exact spot in space	34. point
	A straight path that has one endpoint and extends forever in the opposite direction	35. ray
	Lines that cross at a point	36. intersecting lines
	Lines that do not cross no matter how far they are extended	37. parallel lines
	A straight path between two endpoints	38. line segment
	Lines that cross at $90°$	39. perpendicular lines
	A thin slice of space extending forever in all directions	40. plane
	A straight path that extends forever in both directions	41. line

Section 7: Triangles

Word bank: Acute triangle Right triangle Obtuse triangle Scalene triangle Isosceles triangle Equilateral triangle	A triangle with one angle of $90°$	42. right triangle
	A triangle with all angles less than $90°$	43. acute triangle
	A triangle with no congruent sides	44. scalene triangle
	A triangle with ***at least*** 2 congruent sides	45. isosceles triangle
	A triangle with an angle greater than $90°$	46. obtuse triangle
	A triangle with 3 congruent sides	47. equilateral triangle

Activity 9-2: **Geometry Vocabulary** Name: ____________________

Polygons	Triangles
Regular polygon	Equilateral triangles
Quadrilaterals	Scalene triangles
Pentagons	Isosceles triangles
Hexagons	Acute triangles
Heptagons	Right triangles
Octagons	Obtuse triangles
Nonagons	Rectangles
Decagons	Squares
Circles	Parallelograms
Ovals	Rhombuses
Lines	Trapezoids
Rays	Line segments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V
all sides the same length
W
X
Y
Z
A1
B1
C1
all sides the same length
D1
E1
F1
G1
H1
J1
K1
L1
M1
N1
O1
P1
Q1
R1
S1
T1

Activity 9-3: **Size and Shape** Name:

Properties of Similar and Congruent Shapes

http://www.virtualnerd.com/geometry/similarity/polygons/similar-figures-missing-measurement-example

http://www.virtualnerd.com/geometry/similarity/triangles/indirect-measurement-example

http://www.purplemath.com/modules/ratio6.htm

http://www.mathwarehouse.com/geometry/similar/triangles/sides-and-angles-of-similar-triangles.php

Similar figures are the same shape but may be different sizes.

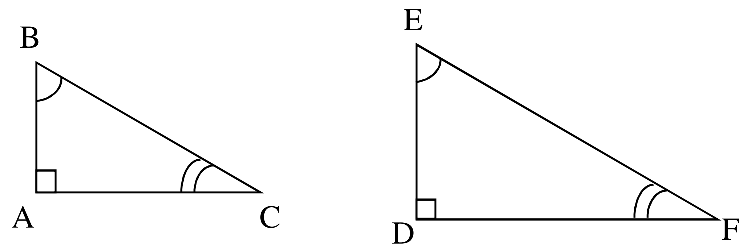
$\Delta ABC \sim \Delta DEF$

Corresponding angles are congruent.

$\angle A \cong \angle D \quad \angle B \cong \angle E \quad \angle C \cong \angle F$

Corresponding side lengths are proportional.

$\frac{AB}{DE} = \frac{BC}{EF} \qquad \frac{AC}{DF} = \frac{AB}{DE} \qquad \frac{BC}{EF} = \frac{AB}{DF}$



Congruent figures have the exact same shape and size.

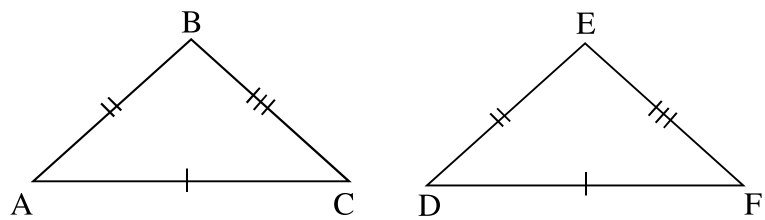
$\Delta ABC \cong \Delta DEF$

Corresponding angles are congruent.

$\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$



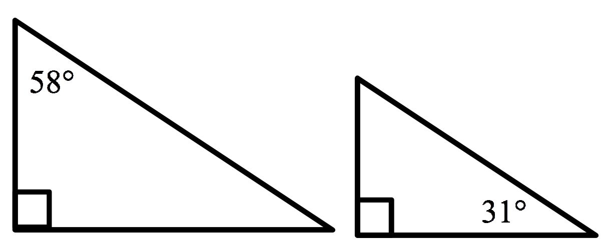
Corresponding side lengths are congruent.

$\overline{AB} \cong \overline{DE}$

$\overline{AC} \cong \overline{DF}$

$\overline{BC} \cong \overline{EF}$

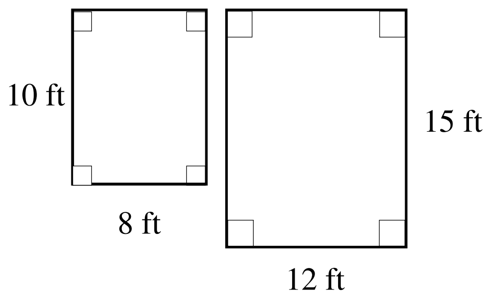
Is there enough information to prove whether these triangles are similar? If so, are they?



Activity 9-4: **Similar Polygons** Name: ______________________

Set up a proportion to tell whether each pair of polygons is similar.

1.



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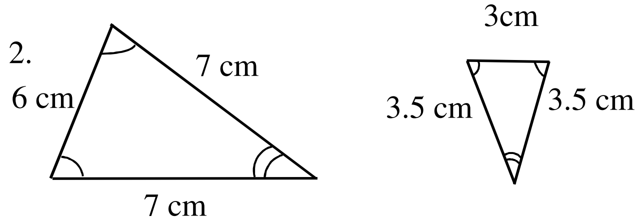

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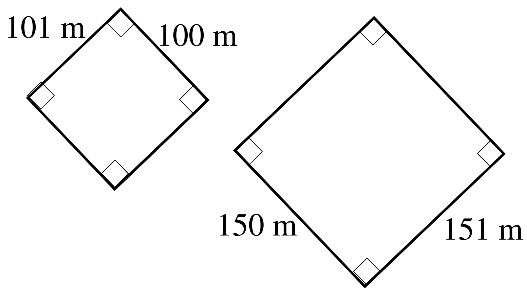

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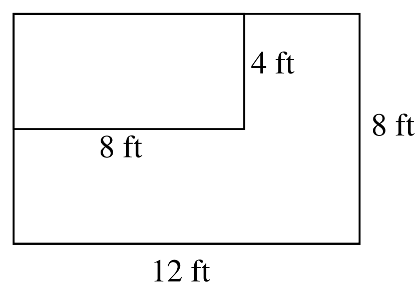

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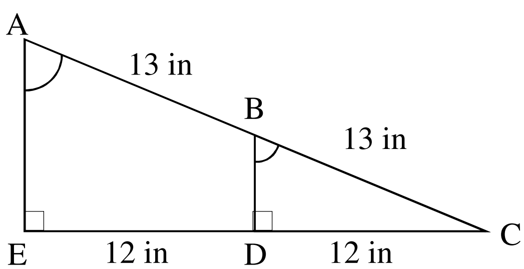
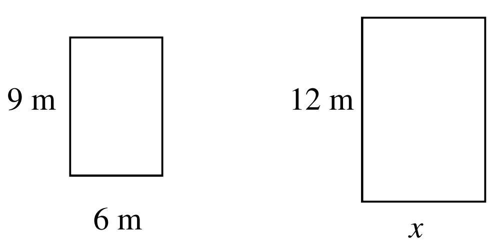


For each pair of similar figures write a proportion and use the proportion to find the length of *x*.
Use a separate sheet of paper.

6.



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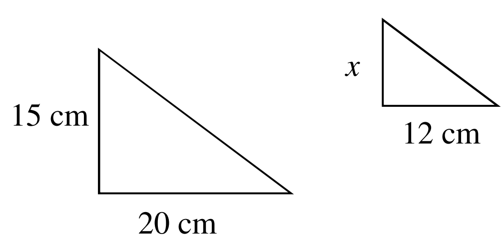

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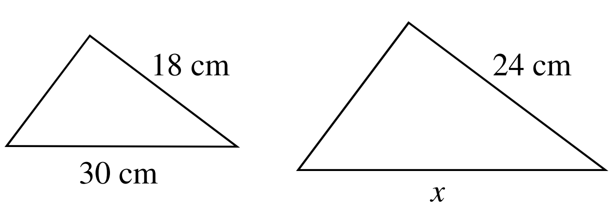

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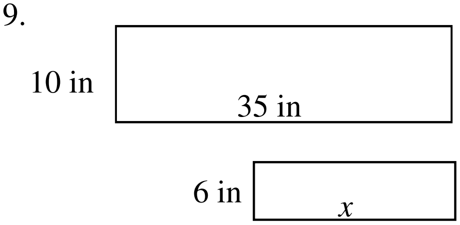
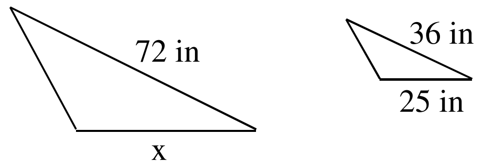


Activity 9-5: **Proportions with Similar Figures** Name: ______________________

For each pair of similar figures write a proportion and use the proportion to find the length of *x*. Use a separate sheet of paper.

1.



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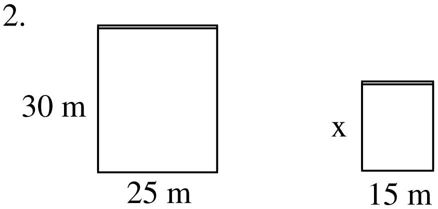

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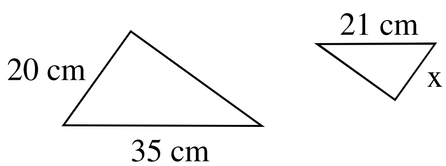

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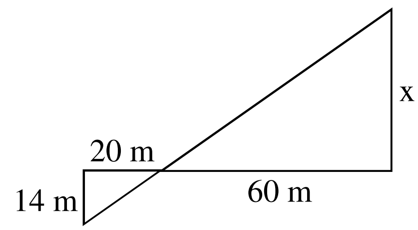

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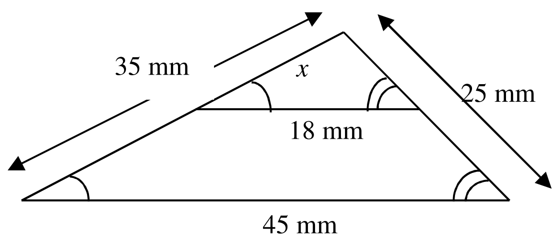

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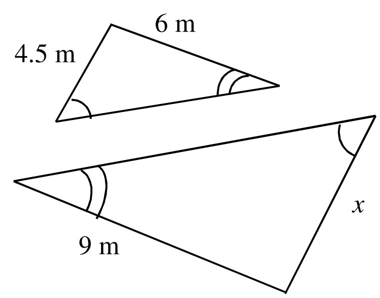

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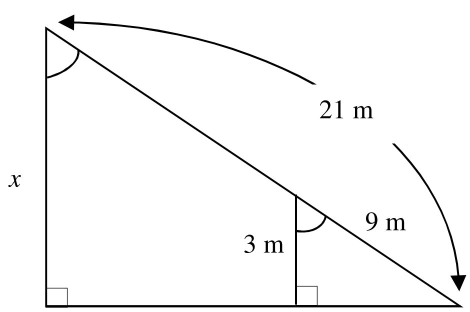

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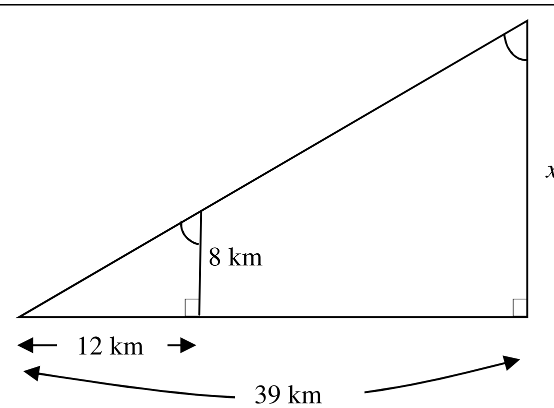
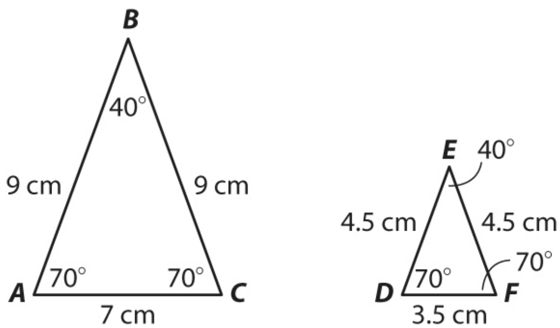


9.	A flagpole casts a shadow 22 ft long. If a 4 ft tall pole casts a shadow 8.8 ft long at the same time of day, how tall is the flagpole?	
10.	A photograph is 25 cm wide and 20 cm high. It must be reduced to fit a space that is 8 cm high. Find the width of the reduced photograph.	
11.	Michael wants to find the length of the shadow of a tree. He first measures a fencepost that is 3.5 feet tall and its shadow is 10.5 feet long. Next, Michael measures the height of the tree, and finds it is 6 feet tall. How long is the shadow of the tree?	

Activity 9-6: **Similar Shapes** Name:

Tell whether the shapes below are similar. Explain your answer.

1.



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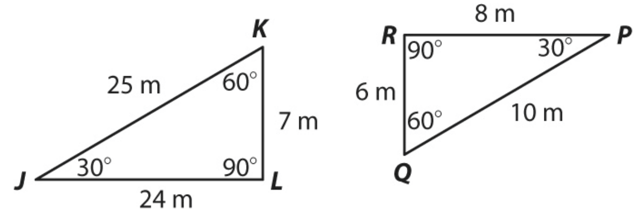

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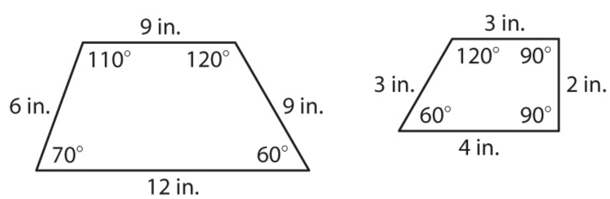

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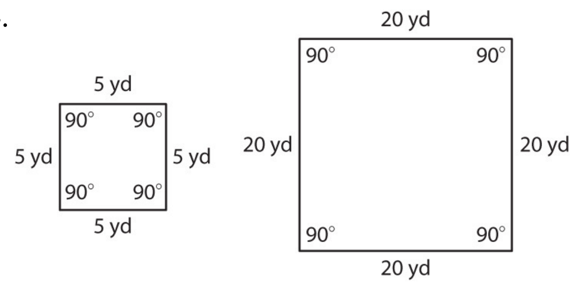
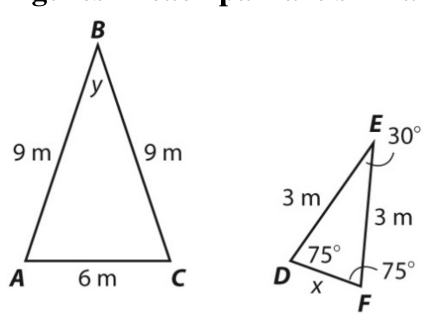


Solve

5.	A rectangle made of square tiles measures 8 tiles wide and 10 tiles long. What is the length in tiles of a similar rectangle 12 tiles wide?	
6.	A computer monitor is a rectangle. Display A is 240 pixels by 160 pixels. Display B is 320 pixels by 200 pixels. Is Display A similar to Display B? Explain.	

The figures in each pair are similar. Find the unknown measures.

7.



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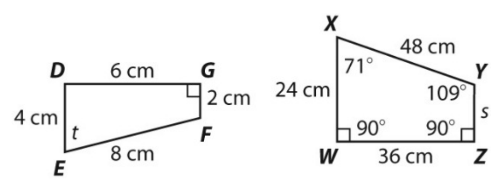

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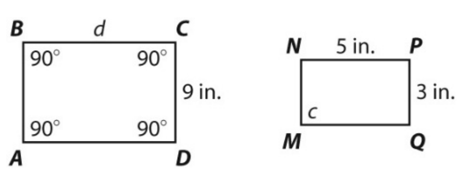
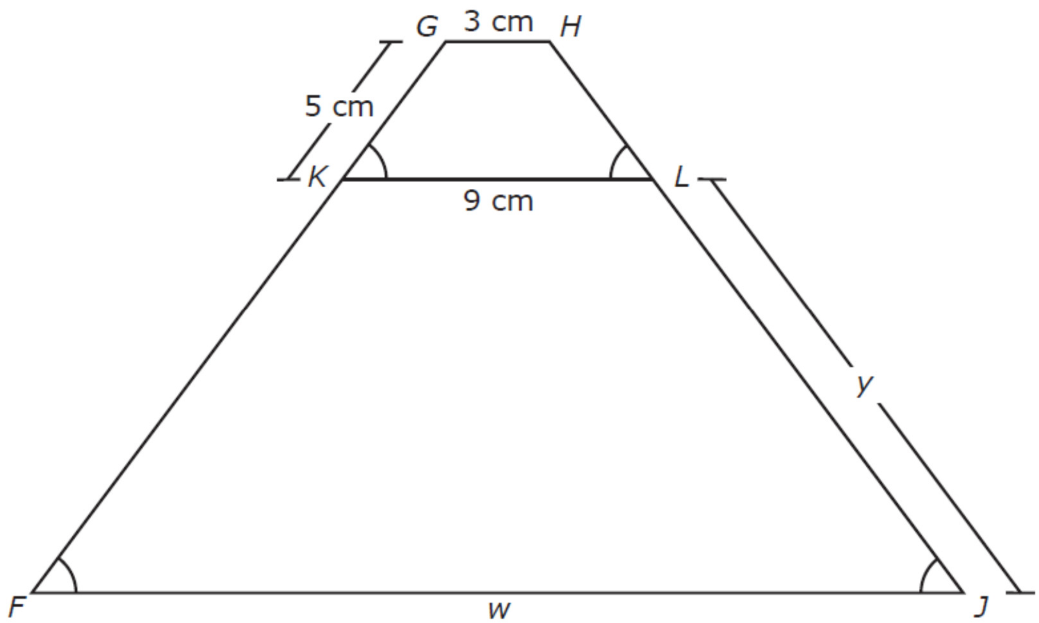


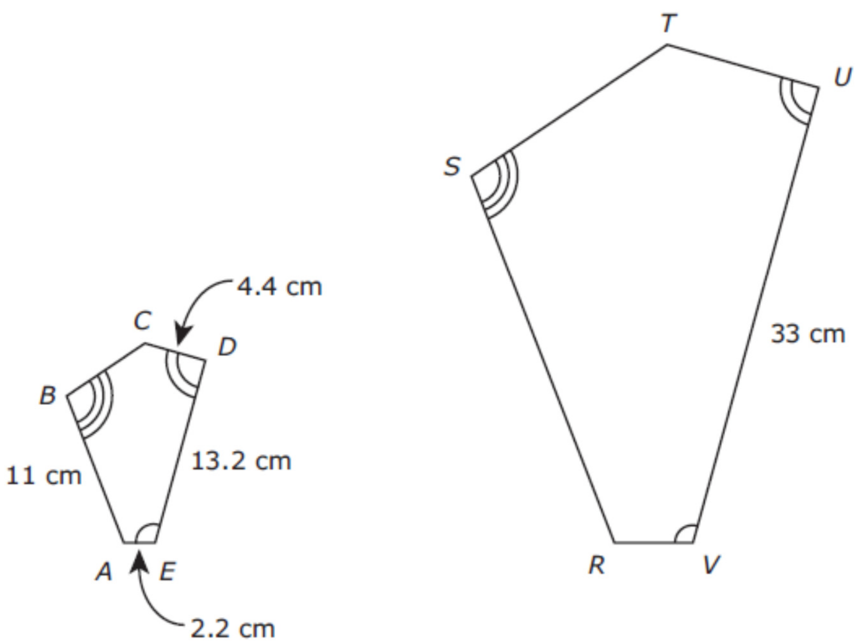
Activity 9-7**: Similar Shapes** Name:

Janelle drew $\overline{KL}$ in isosceles trapezoid FGHJ to create similar trapezoids FKLJ and KGHL.



Based on the given information, what are the values of *y* and *w* in centimeters?

Pentagon *ABCDE* is similar to pentagon *RSTUV*. The perimeter of pentagon *ABCDE* is 36.8 centimeters.



What is the perimeter of pentagon RSTUV?

AMBIGRAMS

A graphic artist named John Langdon began to experiment in the 1970s with a special way to write words as ambigrams. Look at all the examples below and see if you can determine what an ambigram is.

Activity 9-8: **Quadrilaterals** Name: ______________________

Choose ALL, SOME, or NO

1.	All	Some	No	rectangles are parallelograms.
2.	All	Some	No	parallelograms are squares.
3.	All	Some	No	squares are rhombi.
4.	All	Some	No	rhombi are parallelograms.
5.	All	Some	No	trapezoids are rectangles.
6.	All	Some	No	quadrilaterals are squares.
7.	All	Some	No	rhombi are squares.
8.	All	Some	No	parallelograms are trapezoids.
9.	All	Some	No	rectangles are rhombi.
10.	All	Some	No	squares are rectangles.
11.	All	Some	No	rectangles are squares.
12.	All	Some	No	squares are quadrilaterals.
13.	All	Some	No	quadrilaterals are rectangles.
14.	All	Some	No	parallelograms are rectangles.
15.	All	Some	No	rectangles are quadrilaterals.
16.	All	Some	No	rhombi are quadrilaterals.
17.	All	Some	No	rhombi are rectangles.
18.	All	Some	No	parallelograms are rhombi.
19.	All	Some	No	squares are parallelograms.
20.	All	Some	No	quadrilaterals are parallelograms.
21.	All	Some	No	parallelograms are quadrilaterals.
22.	All	Some	No	trapezoids are quadrilaterals.

Solve each riddle.

23.	I am a quadrilateral with two pairs of parallel sides and four sides of the same length. All of my angles are the same measure, too. What am I?	
24.	I am a quadrilateral with two pairs of parallel sides. All of my angles are the same measure, but my sides are not all the same length. What am I?	
25.	I am a quadrilateral with exactly one pair of parallel sides. What am I?	
26.	I am a quadrilateral with two pairs of parallel sides. What am I?	

GEOMETRIC TRANSFORMATIONS

Translation – Slide
Reflection – Flip
Rotation – Turn
Dilation – Enlargement or Reduction

Preimage – the shape before it is transformed
Image – the shape after it is transformed

Translation	$(x, y) \to (x+a, y+b)$

Reflection over the y-axis	$(x, y) \to (-x, y)$
Reflection over the x-axis	$(x, y) \to (x, -y)$

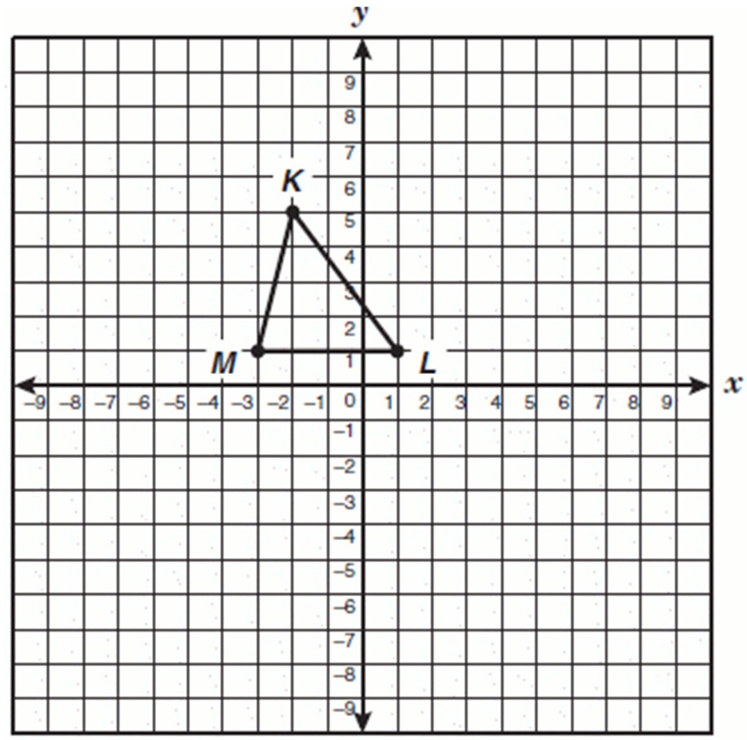
All rotations below are centered about the origin.		
Rotation $90°$ clockwise	$(x, y) \to (y, -x)$	Rotation $270°$ counterclockwise
Rotation $180°$ clockwise	$(x, y) \to (-x, -y)$	Rotation $180°$ counterclockwise
Rotation $270°$ clockwise	$(x, y) \to (-y, x)$	Rotation $90°$ counterclockwise

All dilations below are centered about the origin.	
Dilation	$(x, y) \to (kx, ky)$ k is called the scale factor $k < 1$ means reduction $k > 1$ means enlargement

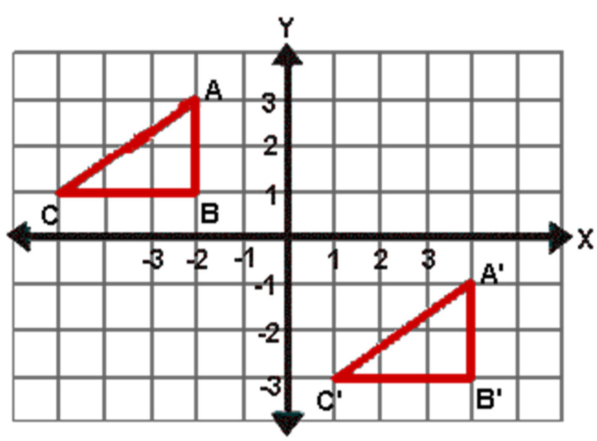
	Translation	**Rotation**	**Reflection**	**Dilation**
Changes Orientation		✓	✓	
Changes Location	✓	✓	✓	✓
Changes Size				✓

Activity 9-9: **Translations** Name:

Consider the triangle shown on the coordinate plane.

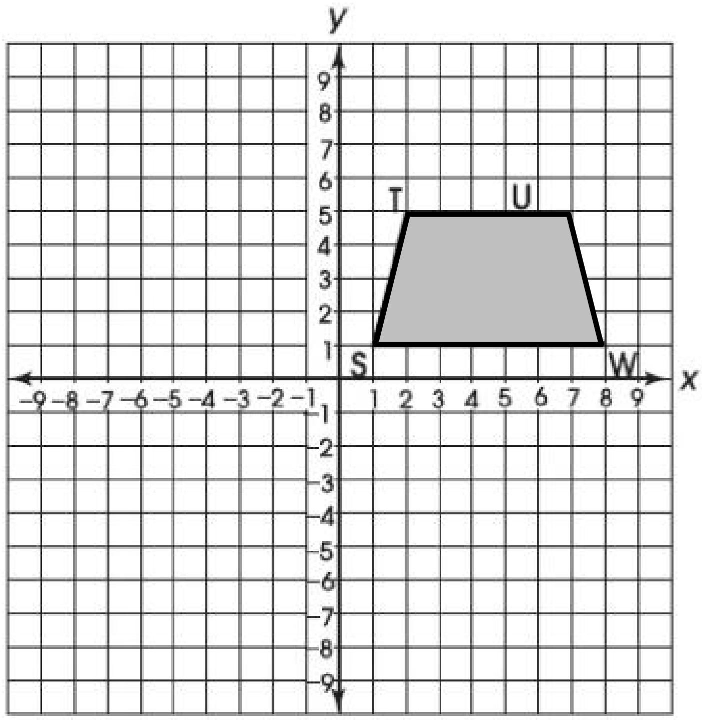


1.	Record the coordinates of the vertices of the triangle.	
2.	Translate the triangle down 2 units and right 5 units. Graph the translation.	
3.	A symbolic representation for the translated triangle would be: $(x, y) \rightarrow (x+5, y-2)$	



4.	Write a verbal description of the translation.	
5.	Describe the translation above using symbolic representation.	

Activity 9-10: **Translations** Name: ______________________



Determine the coordinates of the vertices for each image of trapezoid STUW after each of the following translations in performed.

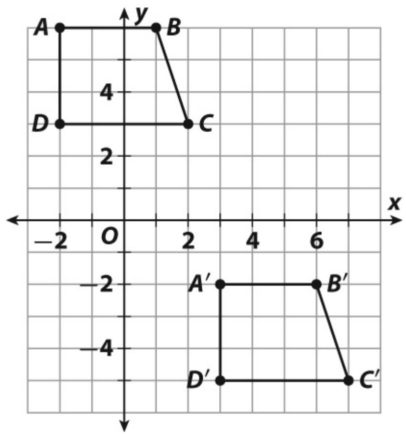
1.	3 units to the left and 3 units down	
2.	$(x, y) \rightarrow (x, y-4)$	
3.	$(x, y) \rightarrow (x-2, y+1)$	
4.	$(x, y) \rightarrow (x-4, y)$	
5.	Find a single transformation that has the same effect as the composition of translations $(x, y) \rightarrow (x-2, y+1)$ followed by $(x, y) \rightarrow (x+1, y+3)$.	

6.	Draw triangle ABC at points $(-1,6), (-4,1), (1,3)$.
7.	Draw triangle $A'B'C'$ is located at $(5,2), (2,-3), (7,-1)$
8.	Write a symbolic representation for the translated triangle compared to the original.
9.	Write a description (in words) of this translation.

10.	Connie translated trapezoid $RSTU$ to trapezoid $R'S'T'U'$. Vertex S was at (-5,-7). If vertex S' is at (-8,5), write a description of this translation.	Move each vertex ___ units to the _________ and ___ units _________.

Activity 9-11**: Properties of Translations** Name: ____________________

Describe the translation that maps point A to point A'.

1. 

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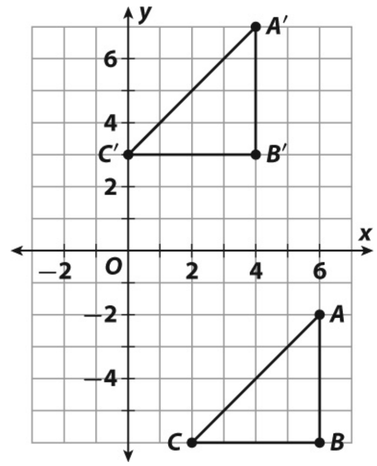
2. 

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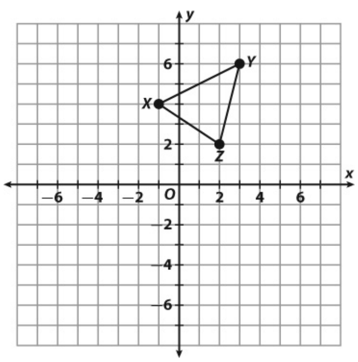
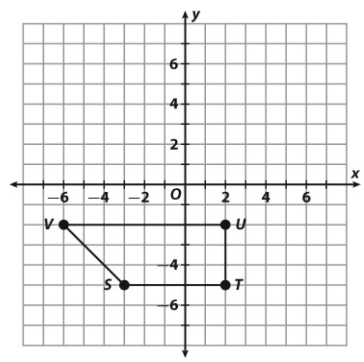
Draw the image of the figure after each translation.

3. 3 units left and 9 units down



4. 3 units right and 6 units up



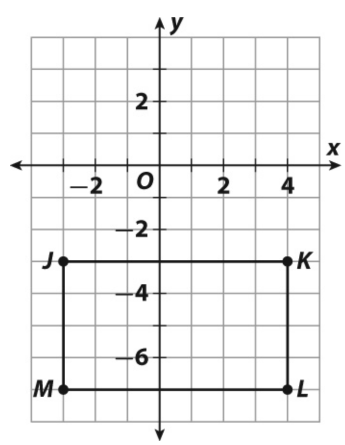
5. a. Graph rectangle $J'K'L'M'$, the image of rectangle $JKLM$, after a translation of 1 unit right and 6 units up.
   b. Find the area of each rectangle.

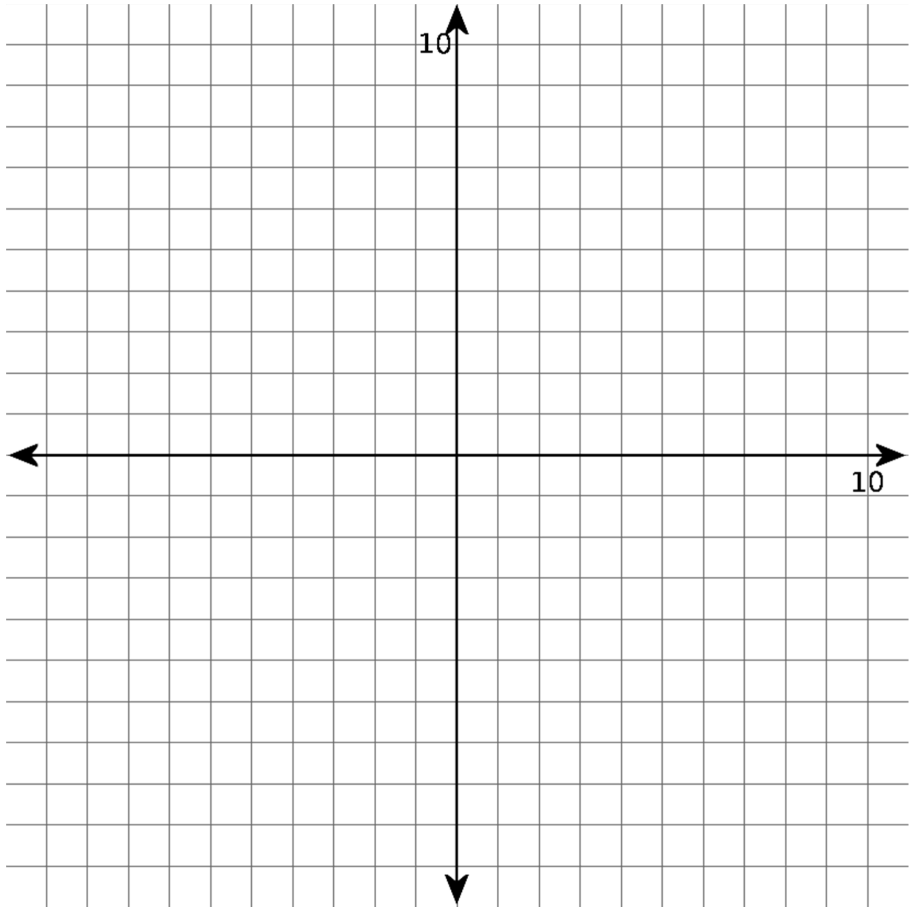
   ____

   c. Is it possible for the area of a figure to change after it is translated? Explain.

   ____

   ____

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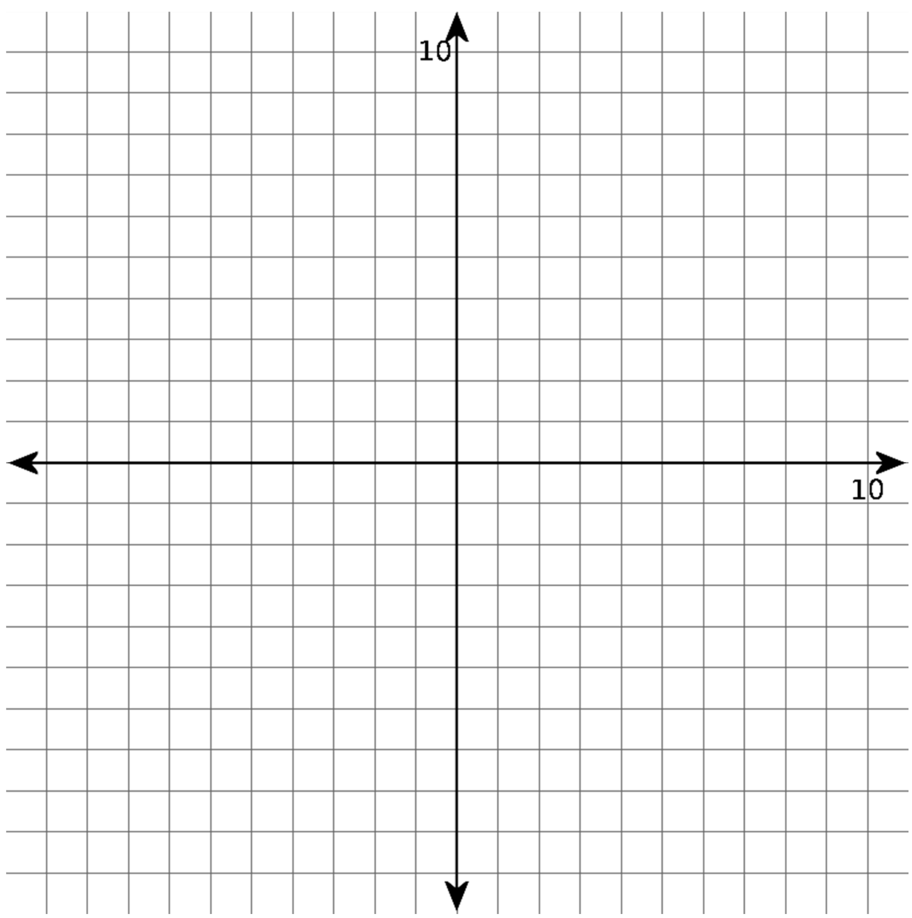
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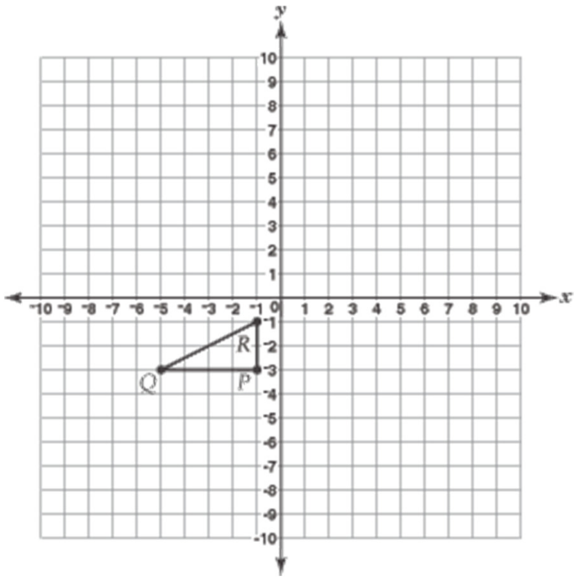
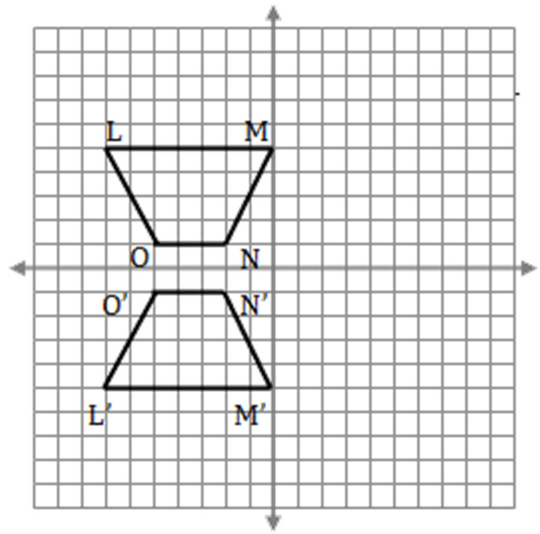
Activity 9-12: **Reflections** Name: ______________________





Use the graph on the left to answer the following questions.

1.	Record the coordinates of the vertices of triangle PQR.	
2.	Sketch the reflection of triangle PQR over the y-axis.	
3.	A symbolic representation for the reflected triangle would be: $(x, y) \rightarrow (-x, y)$	

Use the graph on the right to answer the following questions.
The line over which an object is reflected is called the line of reflection.

4.	What is the line of reflection for the trapezoid above?	
5.	Write a verbal description of the transformation.	
6.	A symbolic representation for the reflected triangle would be: $(x, y) \rightarrow (x, -y)$	

Use a piece of graph paper to draw the following.

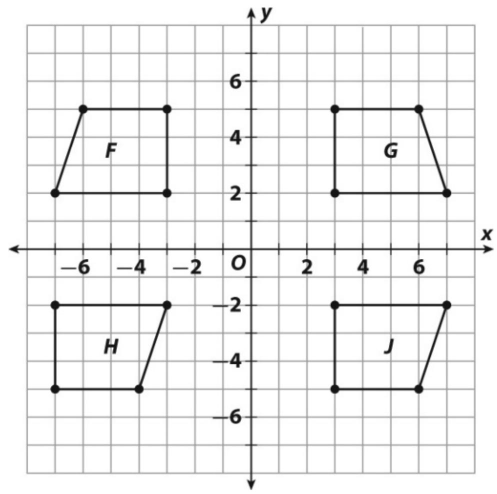
7.	Draw triangle ABC at points $(-4,1),(-1,3),(-5,6)$.
8.	Reflect triangle ABC across the y-axis. List the new vertices $A'B'C'$. Write a symbolic representation for the reflected triangle compared to the original.
9.	Reflect triangle ABC across the x-axis. List the new vertices $A''B''C''$. Write a symbolic representation for the reflected triangle compared to the triangle $A'B'C'$.
10.	If point Q(6,-2) is reflected across the x-axis, what will be the coordinates of point Q' ?

Activity 9-13: **Properties of Reflections** Name: ______________________

Use a piece of graph paper to draw the following.

1.	Draw triangle ABC at points $(-1,1),(-3,-2),(2,-3)$.
2.	Reflect triangle ABC across the y-axis. List the new vertices $A'B'C'$. Write a symbolic representation for the reflected triangle compared to the original.
3.	Reflect triangle ABC across the x-axis. List the new vertices $A''B''C''$. Write a symbolic representation for the reflected triangle compared to the triangle ABC.

Use the graph for next questions.



4. Quadrilateral *J* is reflected across the *x*-axis. What is the image of the reflection?

   ____

5. Which two quadrilaterals are reflections of each other across the *y*-axis?

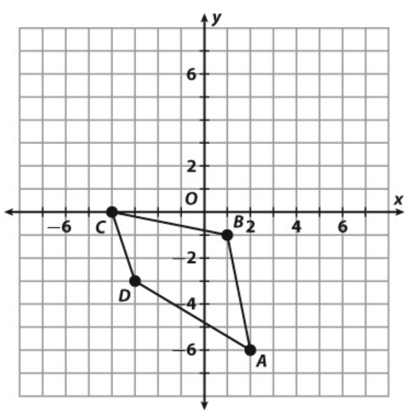
   ____

6. How are quadrilaterals *H* and *J* related?

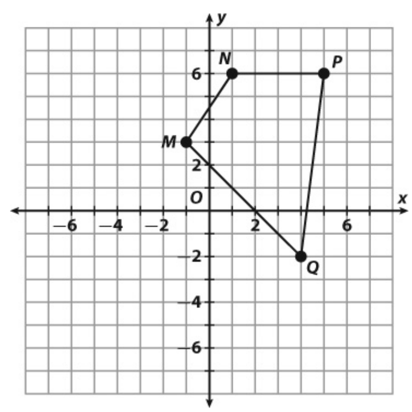
   ____

Draw the image of the figure after each reflection.

7. across the *x*-axis



8. across the *y*-axis



9. a. Graph rectangle *K'L'M'N'*, the image of rectangle *KLMN* after a reflection across the *y*-axis.

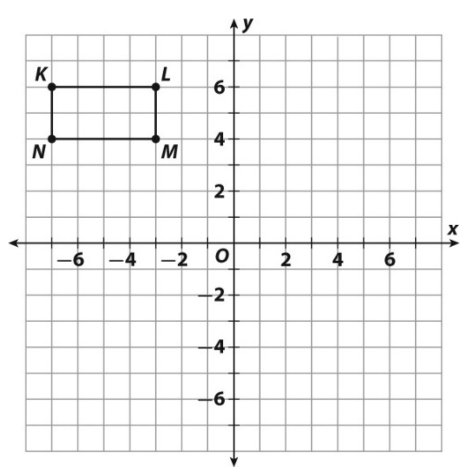
   b. What is the perimeter of each rectangle?

   ____

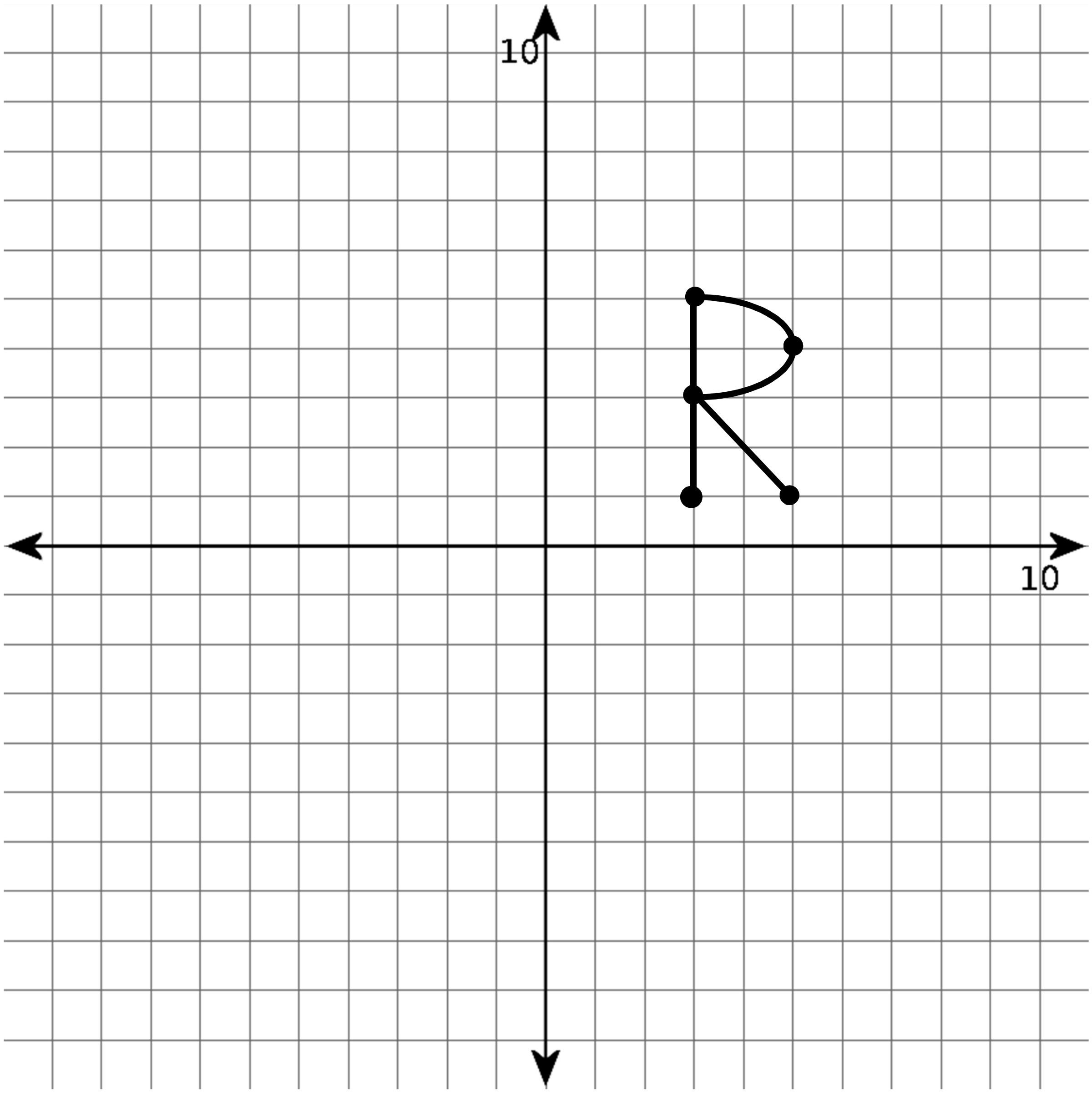
   c. Is it possible for the perimeter of a figure to change after it is reflected? Explain.

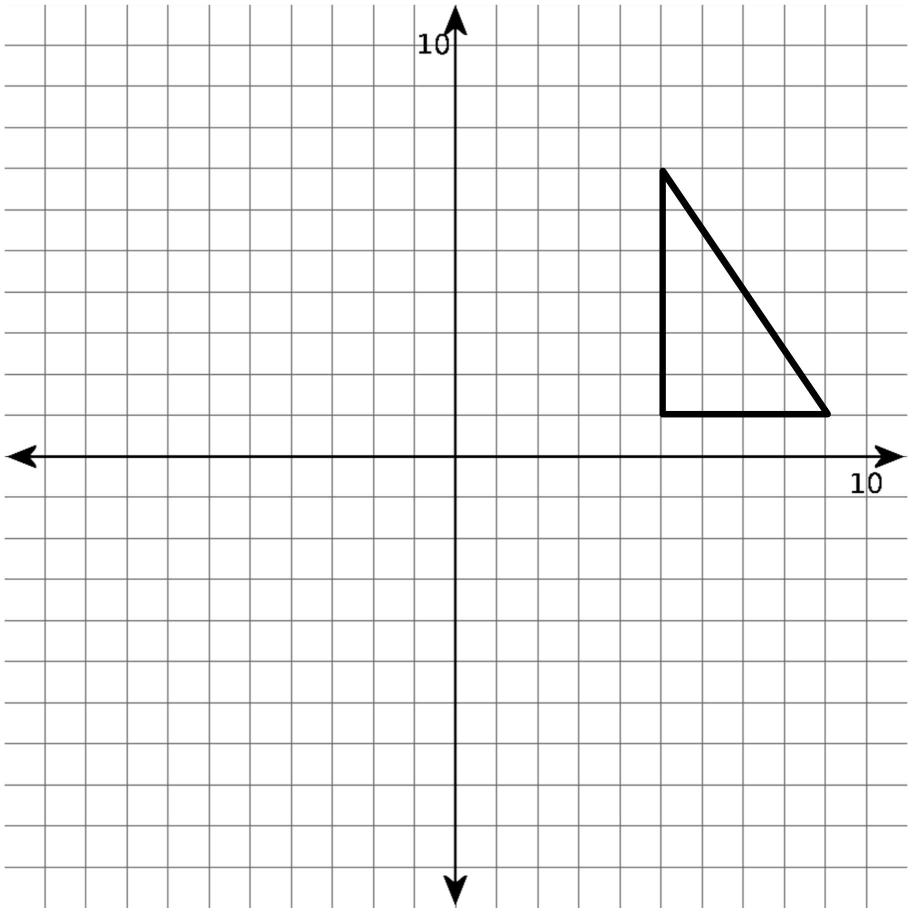
   ____

   ____



https://www.youtube.com/watch?v=7vKxhfPMyAo

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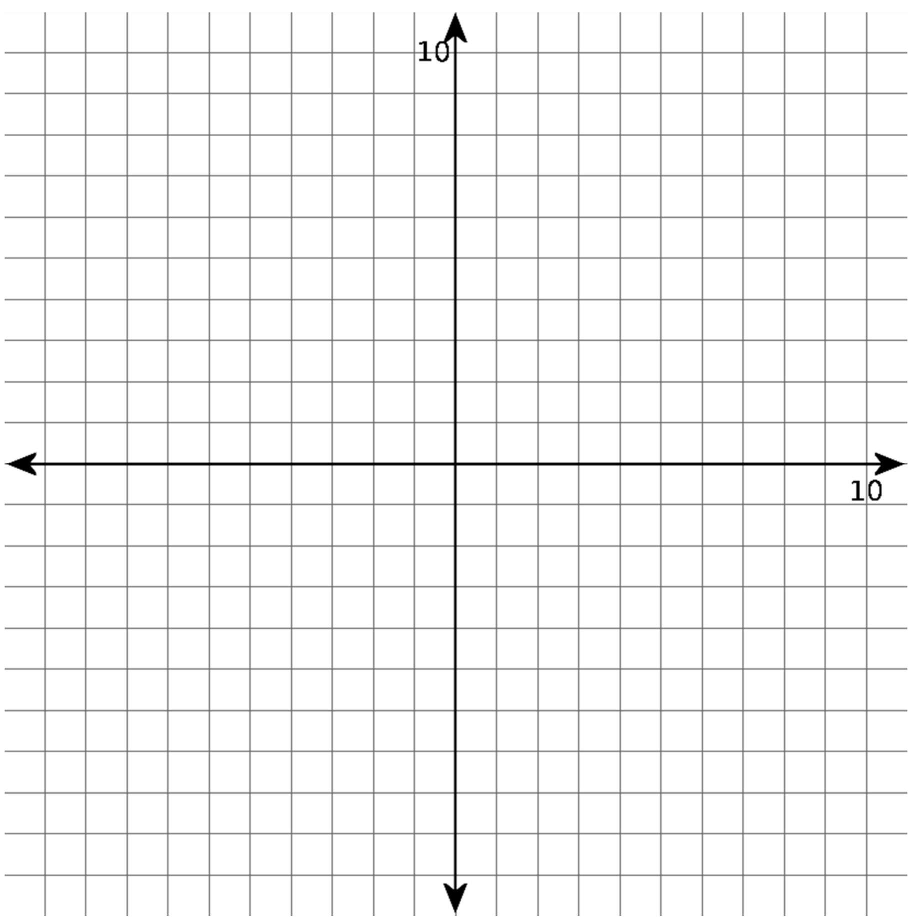
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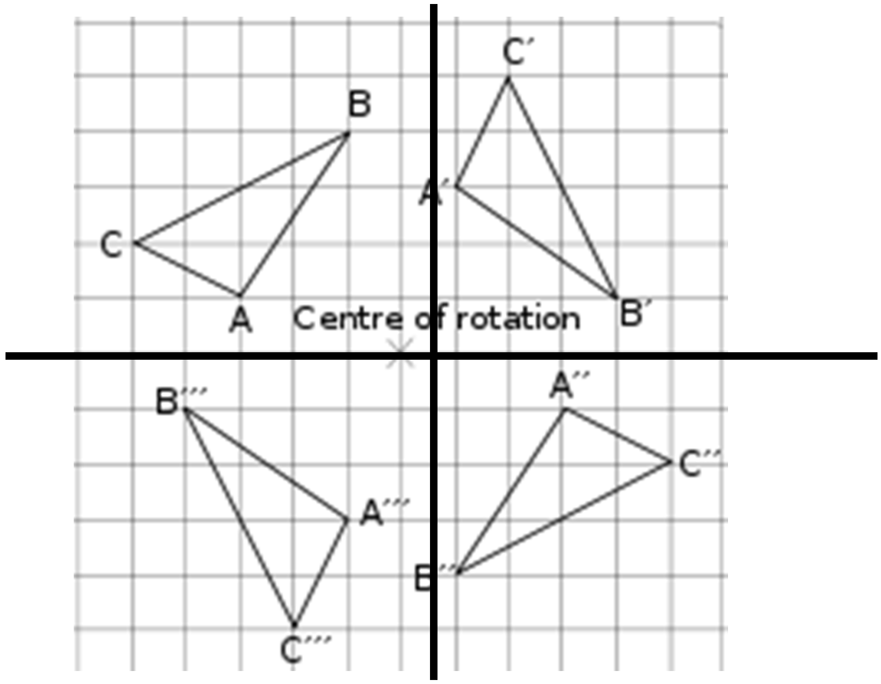
Activity 9-14: **Rotations** Name:

A rotation is a transformation that describes the motion of a figure about a fixed point.

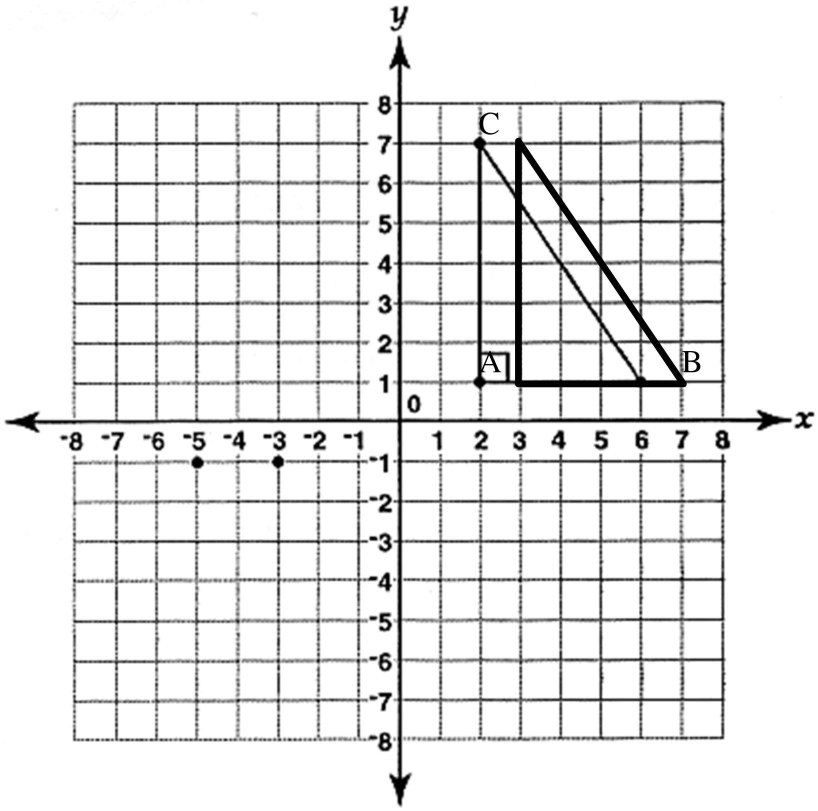


In the table below record the vertices of each triangle.

Triangle	**Three vertices**		
ABC			
$A'B'C'$			
$A''B''C''$			
$A'''B'''C'''$			

1.	Make a conjecture about the changes in the x and y coordinates when a point is rotated clockwise $90°$.
2.	Make a conjecture about the changes in the x and y coordinates when a point is rotated clockwise $180°$.
3.	Make a conjecture about the changes in the x and y coordinates when a point is rotated clockwise $270°$.
4.	What would happen if a shape is rotated $360°$ clockwise about the origin?
5.	What is true about the area of the triangle each time the shape is rotated?

Activity 9-15: **Rotations** Name: ____________________



Starting with the triangle above, rotate the triangle about the origin:

A.	$90°$ counterclockwise
B.	$180°$ counterclockwise
C.	$270°$ counterclockwise

Record the coordinates of the original triangle.

Record the coordinates of the $90°$ counterclockwise rotated circle.

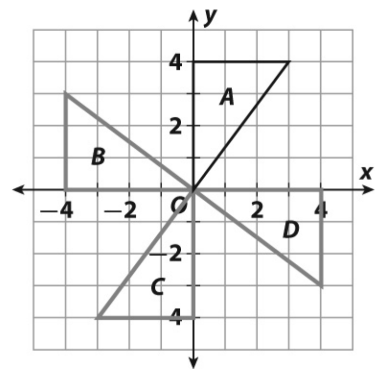
Record the coordinates of the $180°$ counterclockwise rotated circle.

Record the coordinates of the $270°$ counterclockwise rotated circle.

Activity 9-16**: Properties of Rotations** Name: ____________________

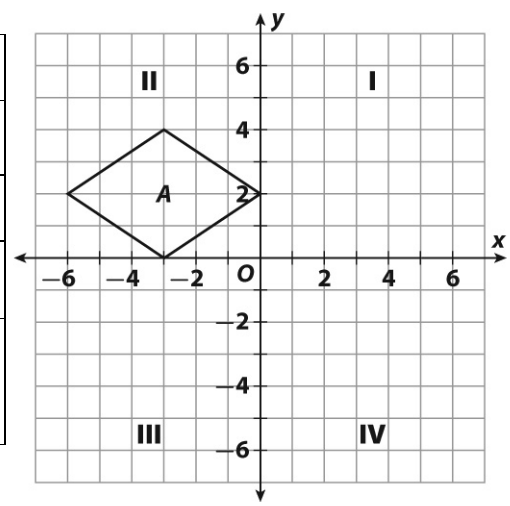
Use the figures at the right for Exercises 1–5. Triangle *A* has been rotated about the origin.

1.	Which triangle shows a 90°counterclockwise rotation?	
2.	Which triangle shows a 180°counterclockwise rotation?	
3.	Which triangle shows a 270°clockwise rotation?	
4.	Which triangle shows a 270°counterclockwise rotation?	
5.	If the sides of triangle *A* have lengths of 30 cm,40 cm, and 50 cm, what are the lengths of the sides of triangle *D*?	



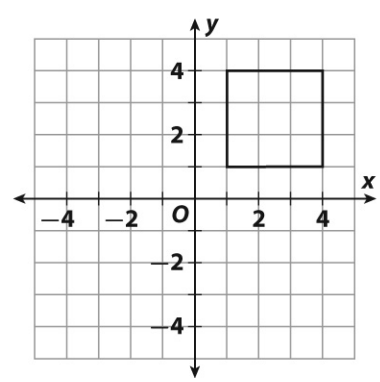
Use the figures at the right for Exercises 6–10. Figure *A* is to be rotated about the origin. Draw all rotated figures on the coordinate plane on the right.

6.	Rotate figure *A* 90° counterclockwise to create figure *B*.	
7.	Rotate figure *A* 270° counterclockwise to create figure *C*.	
8.	Rotate figure *A* 180° clockwise to create figure D.	
9.	If you rotate figure *A* 360° clockwise, what quadrant will the image be in?	
10.	If the measures of two angles in figure *A* are 60° and 120°, what will the measure of those two angles be in the rotated figure?	



Use the grid at the right for Exercises 11–12.

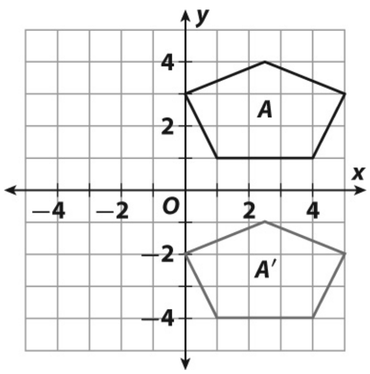
11.	Draw a square to show a rotation of 90° clockwise about the origin of the given square in quadrant I.	
12.	What other transformation would result in the same image as you drew in Exercise 11?	



Activity 9-17**: Algebraic Representations of Transformations** Name: ________________________

Write an algebraic rule to describe each transformation of figure *A* to figure *A′*. Then describe the transformation.

1.



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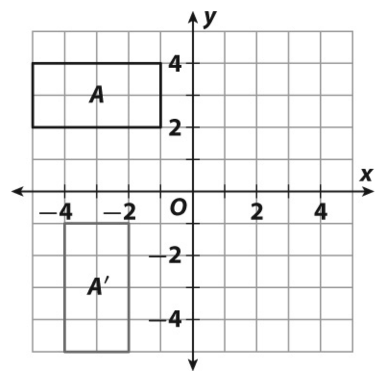
2.



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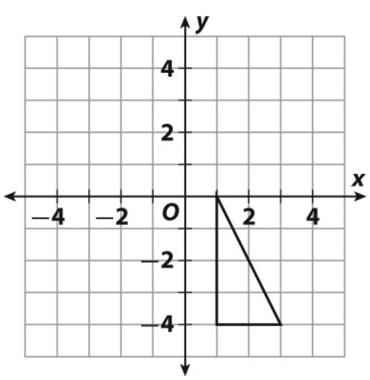
Use the given rule to graph the image of each figure. Then describe the transformation.

3. $(x, y) \rightarrow (-x, y)$



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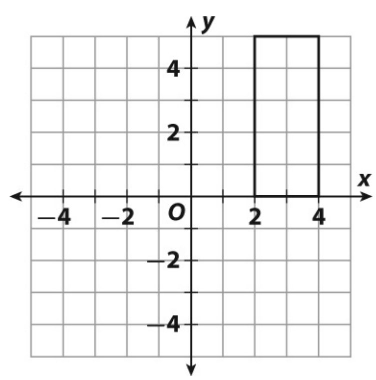
4. $(x, y) \rightarrow (-x, -y)$



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Solve.

5.	Triangle *ABC* has vertices *A*(2, –1), *B*(–3, 0), and *C*(–1, 4). Find the vertices of the image of triangle *ABC* after a translation of 2 units up.	
6.	Triangle *LMN* has *L* at (1, –1) and *M* at (2, 3). Triangle *L′M′N′* has *L′* at (–1, –1) and *M′* is at (3, –2). Describe the transformation.	

Activity 9-18: **Dilations** Name: ____________________

When two figures satisfy at least one of the following conditions, then they are similar:

- The corresponding angles are congruent.
- The corresponding sides are proportional.

The ratio of the corresponding sides of similar triangles is called the scale factor.

A **dilation** is a transformation where the image is similar to the preimage (image before the transformation). The center of dilation is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move.

Figure 1 – Example of a dilation

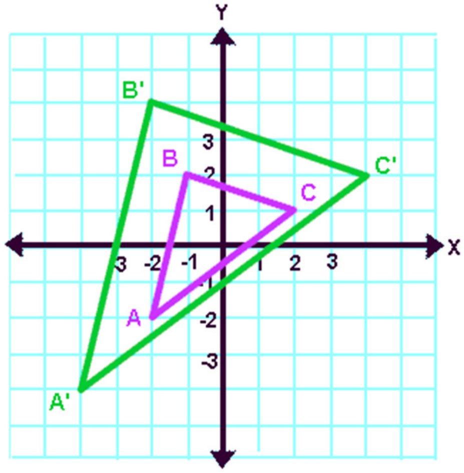
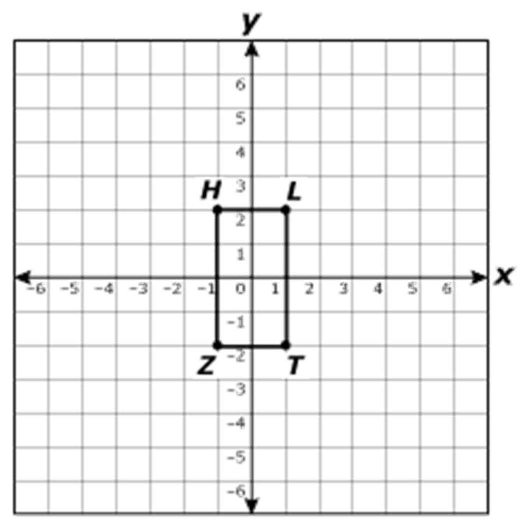


Figure 2



When the origin is the center of dilation you can determine the new coordinates for a dilation by a factor of 2 by: $(x, y) \rightarrow (2x, 2y)$

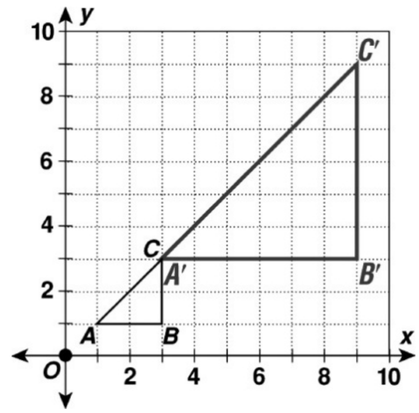
When the scale factor is greater than 1, the dilation is called an **enlargement**.
When the scale factor is between 0 and 1, the dilation is called a **reduction**.

Use Figure 2 above to answer the following questions.

1.	What is the area of $HLTZ$?	
2.	$HLTZ$ is dilated by a factor of 2. Draw the new $H'L'T'Z'$.	
3.	What is the area of $H'L'T'Z'$?	
4.	$HLTZ$ is dilated by a factor of $\frac{1}{2}$. Draw the new $H''L''T''Z''$.	
5.	What is the area of $H''L''T''Z''$?	
6.	How does the dilation factor affect the new area of the rectangles?	

Activity 9-19: **Properties of Dilations** Name:

Use triangles *ABC* and *A'B'C'* for Exercises 1–4.



1. Use the coordinates to find the lengths of the sides.

   Triangle *ABC*: *AB* = ____ ; *BC* = ____

   Triangle *A'B'C'*: *A'B'* = ____ ; *B'C'* = ____

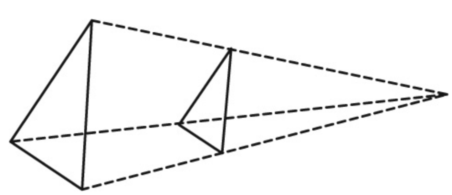
2. Find the ratios of the corresponding sides.

   $\dfrac{A'B'}{AB} = \underline{\quad\quad} = \underline{\quad\quad}$ $\qquad$ $\dfrac{B'C'}{BC} = \underline{\quad\quad} = \underline{\quad\quad}$

3. Is triangle *A'B'C'* a dilation of triangle *ABC*? ____________

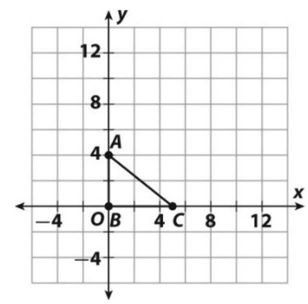
4. If triangle *A'B'C'* is a dilation of triangle *ABC*, is it a reduction or an enlargement? ______________

For Exercises 5–8, tell whether one figure is a dilation of the other or not. If one figure is a dilation of the other, tell whether it is an enlargement or a reduction. Explain your reasoning.

5.	Triangle *R'S'T'* has sides of 3 cm, 4 cm, and 5 cm. Triangle *RST* has sides of 12 cm, 16 cm, and 25 cm.	
6.	Quadrilateral *WBCD* has coordinates of *W*(0, 0), *B*(0, 4), *C*(–6, 4), and *D*(–6, 0). Quadrilateral *W'B'C'D'* has coordinates of *W'*(0, 0), *B'*(0, 2), *C'*(–3, 2), and *D'*(–3, 0).	
7.	Triangle *MLQ* has sides of 4 cm, 4 cm, and 7 cm. Triangle *M'L'Q'* has sides of 12 cm, 12 cm, and 21 cm.	
8.	Does the following figure show a dilation? Explain. 	

Activity 9-20: **Algebraic Representations of Dilations** Name: ____________________

Use triangle *ABC* for Exercises 1–4.



1. Give the coordinates of each vertex of ΔABC.

   *A*______________ *B*______________ *C*______________

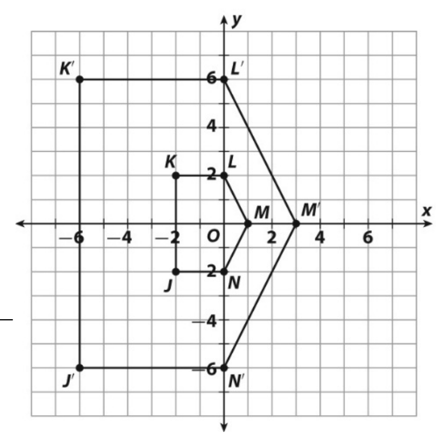
2. Multiply each coordinate of the vertices of ΔABC by 2 to find the vertices of the dilated image $\Delta A'B'C'$.

   *A'*______________ *B'*______________ *C'*______________

3. Graph $\Delta A'B'C'$.
4. Complete this algebraic rule to describe the dilation.

   $(x, y) \rightarrow$ ______________________

Use the figures at the right for Exercises 5–7.



5. Give the coordinates of each vertex of figure *JKLMN*.

   *J*______________ *K*______________ *L*______________

   *M*______________, *N*______________

6. Give the coordinates of each vertex of figure *J'K'L'M'N'*.

   *J'*______________ *K'*______________ *L'*______________

   *M'*______________, *N'*______________

7. Complete this algebraic rule to describe the dilation.

   $(x, y) \rightarrow$ ______________________

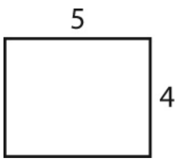
Li made a scale drawing of a room. The scale used was 5 cm = 1 m. The scale drawing is the pre-image and the room is the dilated image.

8.	What is the scale in terms of centimeters to centimeters?	
9.	Complete this algebraic rule to describe the dilation from the scale drawing to the room.	$(x, y) \rightarrow$
10.	The scale drawing measures 15 centimeters by 20 centimeters. What are the dimensions of the room?	

Activity 9-21: **Dilations and Measurement** Name: ____________________

Find the perimeter and area of the original figure and of the image after dilating each figure.

1. Scale factor = 3



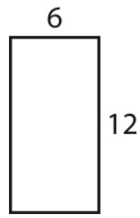
P = ____

P' = ____

A = ____

A' = ____

2. Scale factor = $\frac{1}{3}$



P = ____

P' = ____

A = ____

A' = ____

Solve.

3.	A rectangle is enlarged by a scale factor of 4. The original rectangle is 8.4 cm by 5.3 cm. What are the dimensions of the enlarged rectangle?	
4.	A poster is 16 inches wide by 20 inches long. You use a copier to create a reduction with a scale factor of $\frac{3}{4}$. Will the reduction fit into a frame that is 11 inches by 17 inches? Explain your answer.	

Lincoln School is having a fundraising contest. Each student is to design a school sticker. The designs are to be made to fit a rectangle that is 12 cm by 16 cm. Each design is to have a border.

5.	The winning designs will be made into stickers that are dilated by a scale factor of $\frac{1}{4}$. What will the dimensions of the stickers be? What will the perimeter and the area of the stickers be?	
6.	The winning designs will also be made into posters that are dilated by a scale factor of 6. What will the dimensions of the posters be? What will the perimeter and the area of the posters be?	

Determine if the following scale factor would create an enlargement or reduction.

7. 3.5 8. $\frac{2}{5}$ 9. 0.6 10. $\frac{4}{3}$ 11. $\frac{5}{8}$

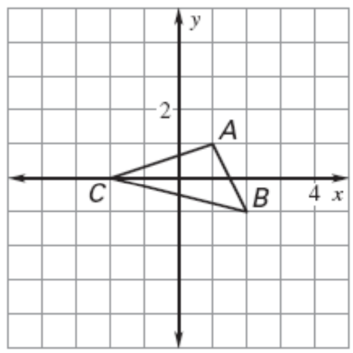
Given the point and its image, determine the scale factor.

12. A(3,6) A' (4.5, 9) 13. G'(3,6) G(1.5,3) 14. B(2,5) B'(1,2.5)

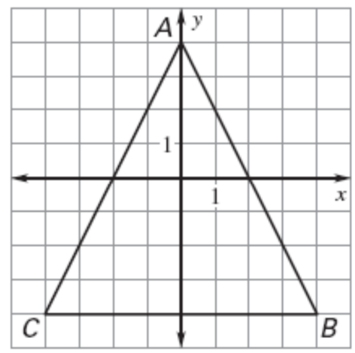
Activity 9-22: **Dilations** Name: ______________________________

Draw a dilation of the figure using the given scale factor, *k*.

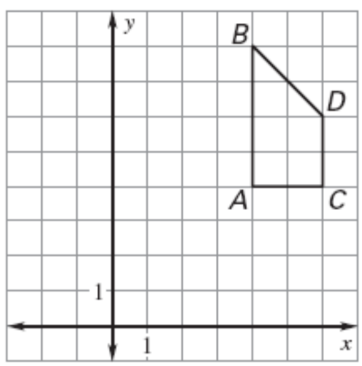
1. $k = 2$



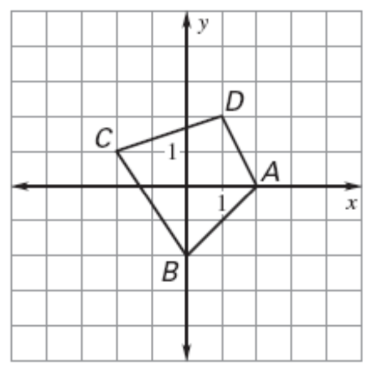
2. $k = \frac{1}{4}$



3. $k = \frac{1}{2}$

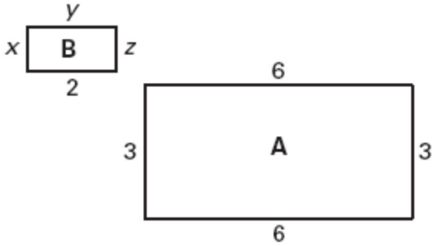


4. $k = 1\frac{1}{2}$

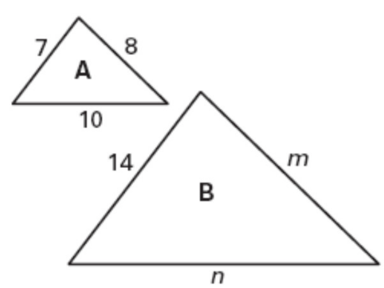


Determine whether the dilation from Figure A to Figure B is a reduction or an enlargement. Then use a proportion to find the values of the variables.

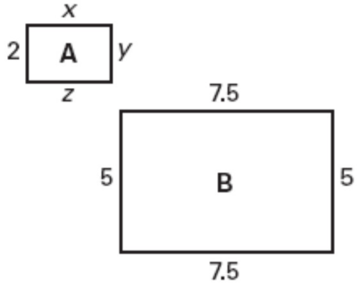
5.



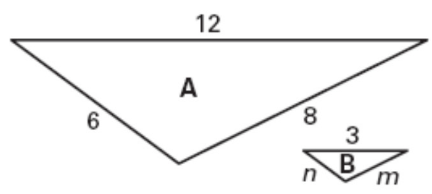
6.



7.



8.



Geometric Transformations: Create An Emblem **Name:**

This project is designed to conclude geometric transformations with students. It includes a review of translations, reflections and rotations on the coordinate grid. Each student has a unique product to make, but they can help each other as needed.

Creating Your Own Emblem

You are going to use your name, a coordinate graph, and some transformations to find your unique emblem.

- First, in the name chart, write the first 6 letters of your first name. If your name is less than 6 letters long, start over on your name.
- Now write in the first 6 letters of your last name. Again if you need more letters start over at the beginning of your name.
- Use the letter-to-number conversion chart to get the coordinates for your original shape. The X coordinate comes from the first name and the Y coordinate comes from the last name. If any ordered pairs duplicate, switch the x- and y-coordinates so that all coordinate pairs are unique.
- Graph your **original** shape on the coordinate grid on the next page by connecting the points along the perimeter of the shape. Make sure your points are connected to form a closed figure. (This may mean that points are not be connected in order.)

Letter	**Value**	**Letter**	**Value**	**Letter**	**Value**
AB	1	IJ	5	QR	9
CD	2	KL	6	ST	10
EF	3	MN	7	UV	11
GH	4	OP	8	WXYZ	12

First	**Last**

Original Figure

	x	***y***	**Coordinate**
A			
B			
C			
D			
E			
F			

Project Directions

Graph your Original Figure

- Using the coordinates from the previous page, graph your original figure. Be sure to rewrite your coordinates in the table provided.
- Make sure ALL shapes you graph form closed figures.

Translation (x – 5, y – 8)

A translation is taking the original image and sliding it without turning it.

- Graph your original shape again.
- Now translate the shape. Find the coordinates for the **image.** Graph the image.

Reflection in the *x*–Axis

A reflection is taking the original image and flipping it along a line of reflection.

- Graph your original shape again.
- Now reflect the shape over the x-axis. Find the coordinates for the **image.** Graph the image.

90° **Clockwise Rotation about the origin**

- Graph your original shape again.
- Rotate the figure **90 degrees clockwise**. Find the coordinates for the **image.** Graph the image.

180° **Rotation about the origin**

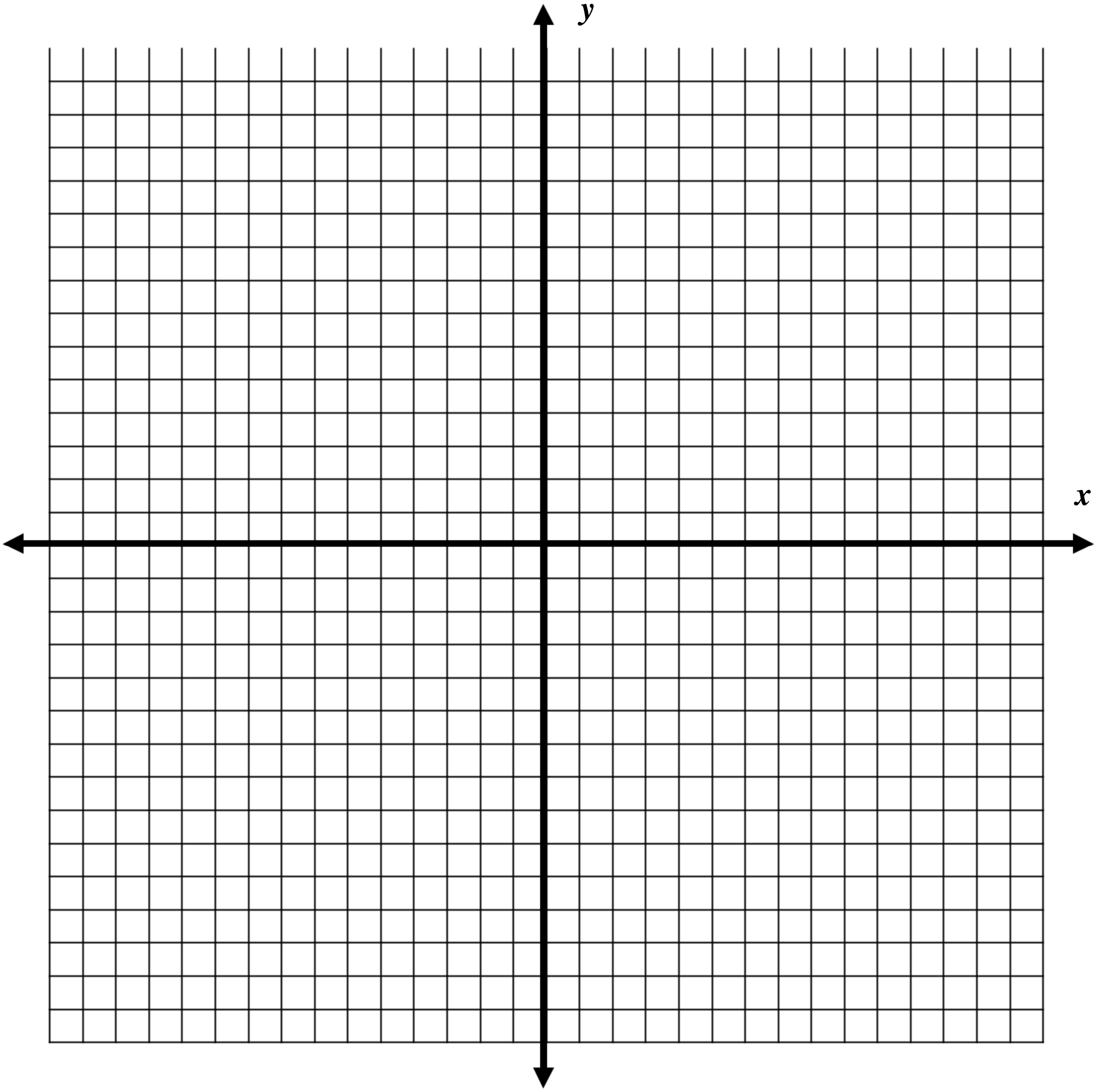
- Graph your original shape again.
- Rotate the figure **180 degrees**. Find the coordinates for the **image.** Graph the image.

Your Emblem

Now to make **your** emblem, which will stand for **you:**

- Graph your original shape.
- Perform a **sequence of two unique transformations** on your original image.
- Your emblem will consist of **THREE** figures:
  - Figure 1: Your original shape (pre-image)
  - Figure 2: Your original shape transformed using a translation (image)
  - Figure 3: The *image* transformed using a reflection or rotation
- Clearly state the sequence of transformations that you used in your emblem.
- Color or decorate. Think of a slogan or motto to go with your emblem.

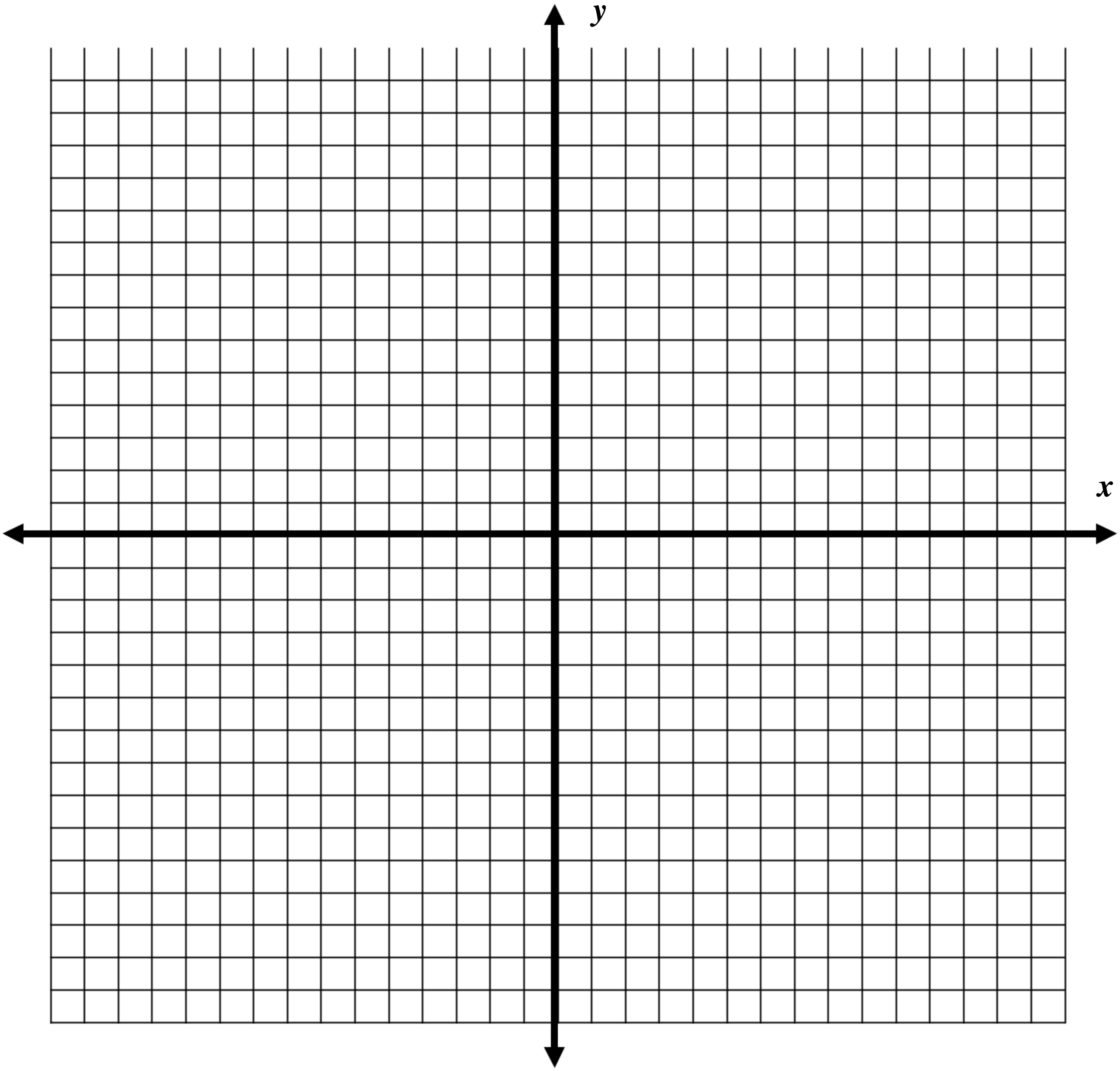
Original Figure



Original Figure

	x	*y*
A		
B		
C		
D		
E		
F		

Transformation #1: Translation $(x - 5, y - 8)$



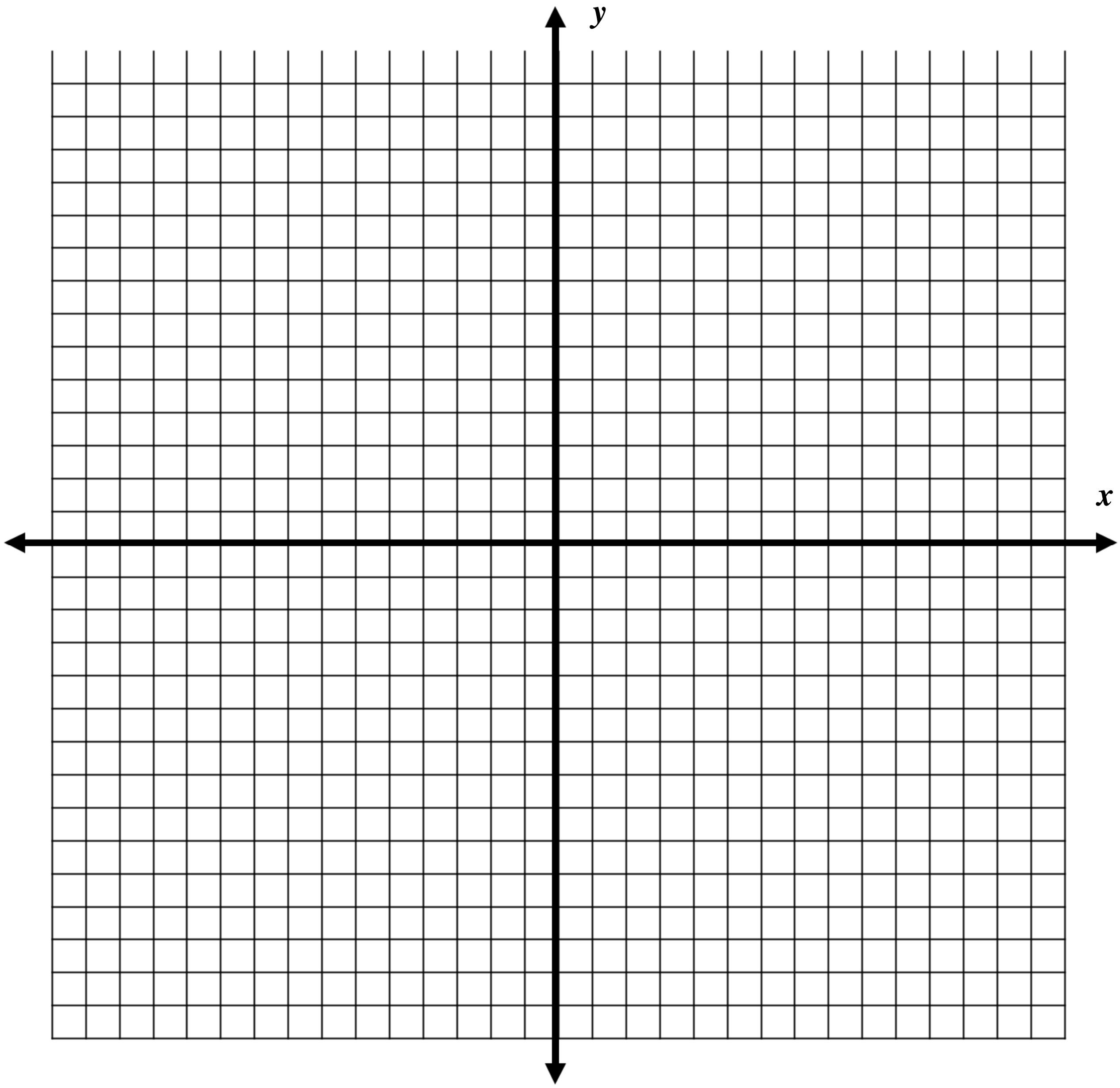
Original Figure

	x	y
A		
B		
C		
D		
E		
F		

Image
Translation $(x - 5, y - 8)$

	x	y
A’		
B’		
C’		
D’		
E’		
F’		

Transformation #2: Reflection across the *x*-Axis



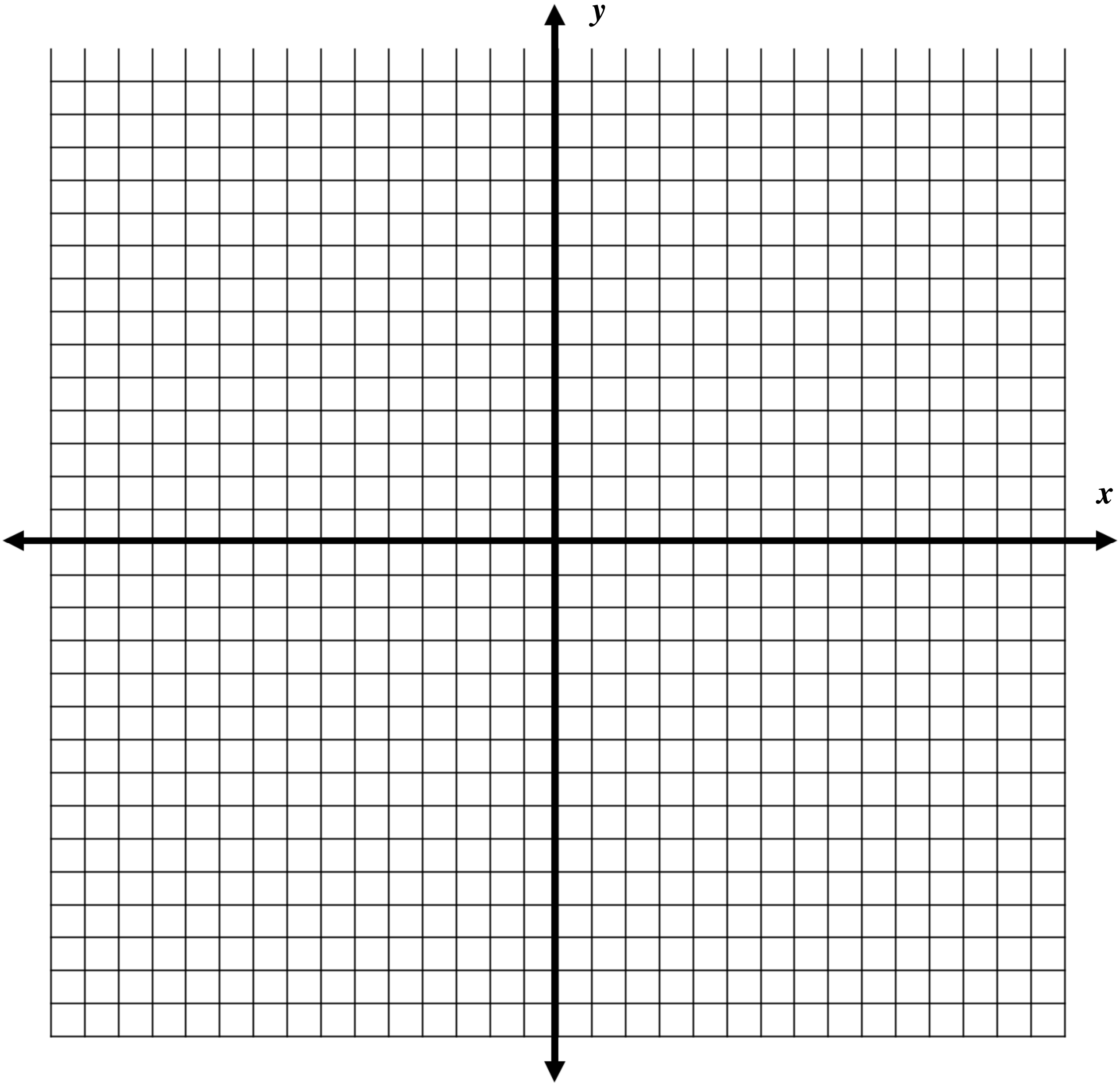
Original Figure

	x	*y*
A		
B		
C		
D		
E		
F		

Image
Reflection over *x*–axis

	x	*y*
A’		
B’		
C’		
D’		
E’		
F’		

Transformation #3: 90° Clockwise Rotation



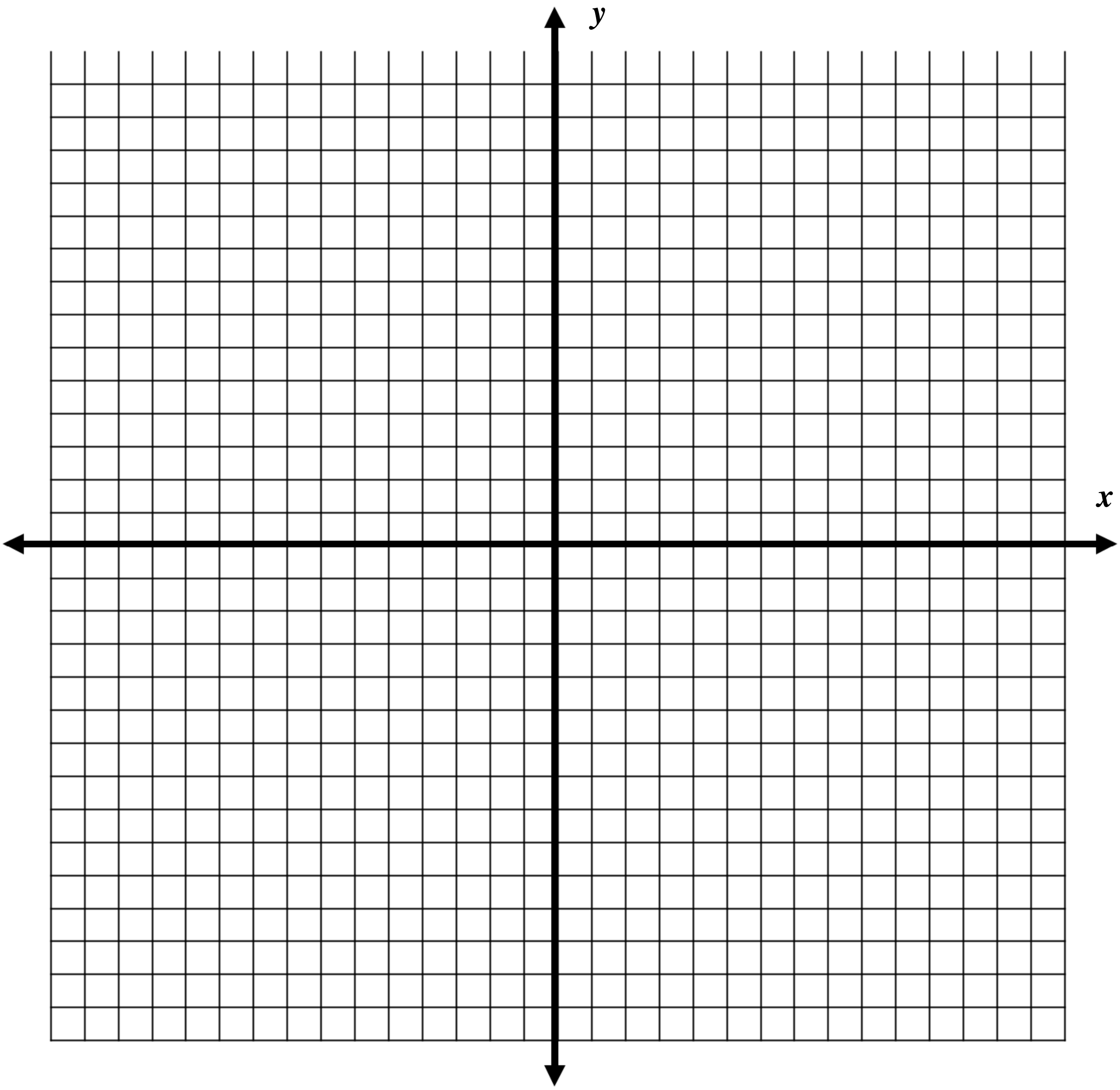
Original Figure

	x	*y*
A		
B		
C		
D		
E		
F		

Image
90 degree
Clockwise Rotation

	x	*y*
A’		
B’		
C’		
D’		
E’		
F’		

Transformation #4: 180^{o} Rotation



Original Figure

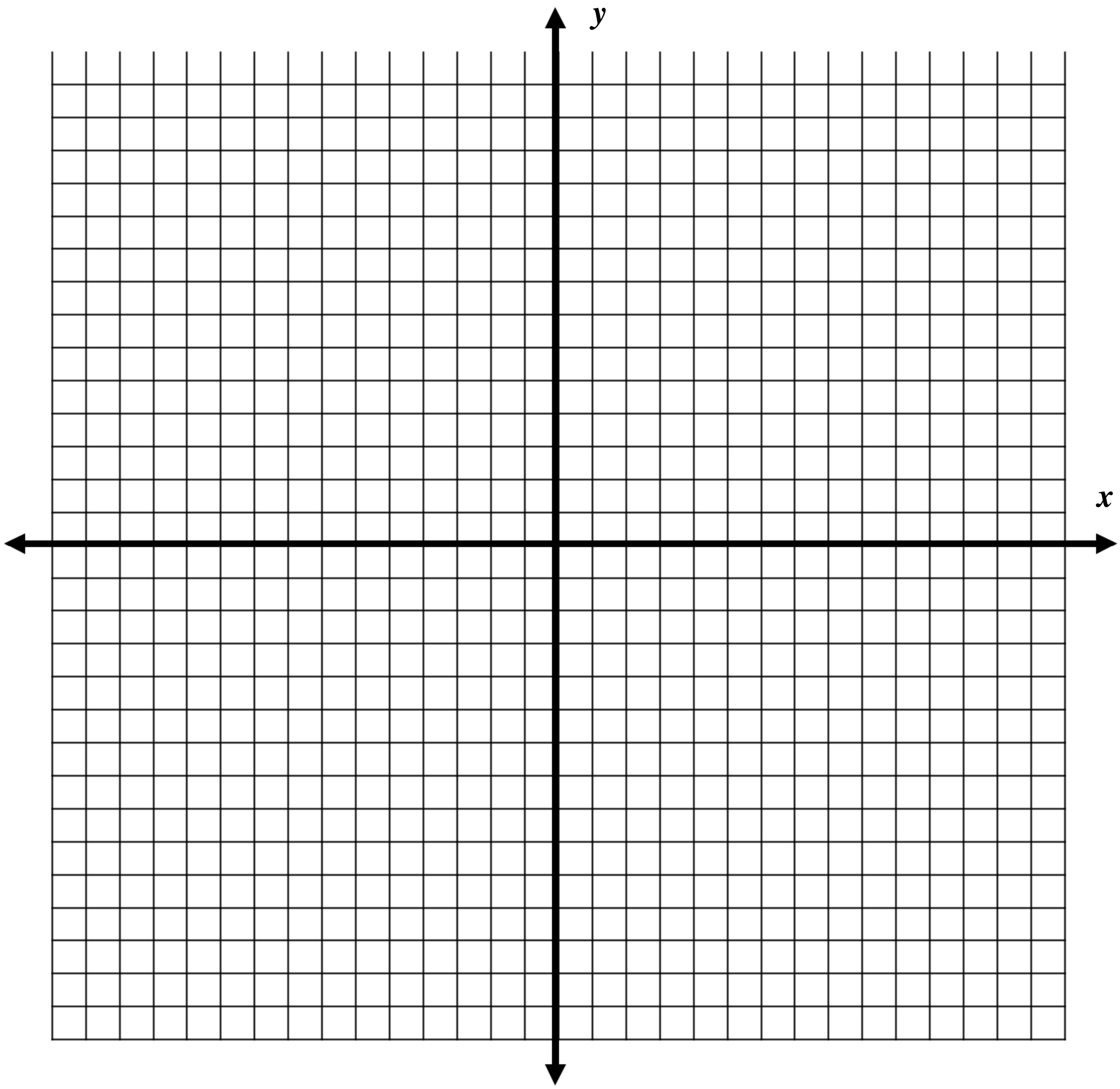
	x	y
A		
B		
C		
D		
E		
F		

Image
180 degree Rotation

	x	y
A’		
B’		
C’		
D’		
E’		
F’		

Created by Lance Mangham, 6^{th} grade math, Carroll ISD

Emblem Name: ______________________________



Sequence of transformations in the emblem:

1. ______________________________

2. ______________________________

MY MOTTO